Intensionality

1. INTENSIONAL PROPOSITIONAL LOGIC (INTPL).

- Intensional PL adds some operators **O** to our standard PL. The crucial property of these operators is that, for any formula ϕ, the truth value that $[\![\mathbf{O}\phi]\!]^s$ yields (in the current situation s) depends not (just) on $[\![\phi]\!]^s$ but on $[\![\phi]\!]^{s'}$ for some other situations s'. This means that the semantics of a language with an expression **O**ϕ involves *quantification over possible situations*, which is what characterizes intensional languages.

- Each operator **O** encodes some quantificational force and ((i)) a restriction specifying the kind of situations it quantifies over. Further restrictions on the set of situations come from (ii) the current evaluation situation s, and –if the operator expresses an attitude (belief, desire, etc.) – from (iii) the holder of the attitude.

(1) Quantificational Force:
a. **It is necessary that** ϕ: "In *all* situations s', $[\![\phi]\!]^{s'}=1$."
b. **It is possible that** ϕ: "In *some* situation s', $[\![\phi]\!]^{s'}=1$."

(2) Restriction on situations by **O**:
a. **It is logically necessary that** ϕ: ALETHIC LOGIC
"In all *logically possible* situations s', $[\![\phi]\!]^{s'}=1$."
b. **It is physical necessity that** ϕ:
"In all possible situations *s' that conform to our physics laws*, $[\![\phi]\!]^{s'}=1$."
c. **It is obligatory that** ϕ: DEONTIC LOGIC
"In all possible situations s' *where all our (legal, moral, etc.) obligations are fulfilled*, $[\![\phi]\!]^{s'}=1$."
d. **It must be the case that** (as opposed to **perharps, may**, etc.) ϕ: EPISTEMIC LOGIC
"In all possible situations s' *that conform to what we know to be the case*, $[\![\phi]\!]^{s'}=1$."

(3) Katherine must be very nice.
a. ✓ Deontic.
b. ✓ Epistemic.

(4) Restriction on situations due to current evaluation situation s:
"Consider a chess player halfway through the game. He knows where all the pieces stand on the board, and he is familiar with the rules of the game, so in principle at least, he is in a position to calculate all of his epistemic alternatives: those positions that can be reached by continuing the game. But the epistemic alternatives will vary with the stage the game has reached. In fact, the set of epistemic alternatives that this player has shrinks as the game progresses, for each move excludes whole branching trees of previously possible developments. Thus the statement **I know that black will not win** may be false in a given context [MR: e.g., at the beginning of the game] (…), while this statement at a later stage in the game becomes true, black having lost all his pieces but the king." (Gamut 2, p. 18)

(5) Holder of the attitude:
a. **It is known to John that** ϕ.
b. **It is known to Peter that** ϕ.

■ To compute the semantics of IntPL formulae, we need (6) [also called "Model"]:

(6) a. a non empty set S of situations.
b. a binary relation R in S encoding the restrictions (i), (ii) and (iii) on S, i.e., a relation S specifying which situations are accessible from each s (and for a given situation holder, if needed).
c. a Lexicon assigning a truth value to every propositional letter p in each situation s.

1.1. Syntax of Modal PL (alethic).

(7) Lexical entries: the propositional letters **p, q, r, s**…, representing atomic statements.

(8) a. Any atomic statement --represented with the letters **p, q, r, s**,…-- is a formula in ModPL.
b. If ϕ is a formula in ModPL, then $\neg\phi$ is a formula in ModPL too.
c. If ϕ and ψ are formulae in ModPL, then $(\phi\wedge\psi)$, $(\phi\vee\psi)$, $(\phi\rightarrow\psi)$, and $(\phi\leftrightarrow\psi)$ are formulae too.
d. If ϕ is a formula in ModPL, then $\Box\phi$ and $\Diamond\phi$ are formulae in ModPL too.
e. Nothing else is a formula in ModPL.

QUESTION 1: Exercise 1 in Gamut p. 21: Translate (9) into ModPL.
(9) a. It is possible that you don't understand me, but it isn't necessary.
b. If it may be raining, then it must be possible that it is raining.
c. It is possible that, if it may be raining, it is raining.
d. If it may be necessary that it is raining, then it must be raining.
e. Maybe it is raining, and perhaps this is necessary. [Two translations]

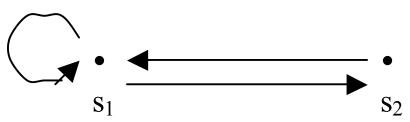
1.2. Semantics of Modal PL (alethic).

(10) For all propositional letters **p** and situations s, $[\![\mathbf{p}]\!]^{s}$ is fixed in the Lexikon (values 1 or 0).

(11) For any ModPL formulae ϕ and ψ, and for any situation s:
a. $[\![\neg\phi]\!]^{s} = 1$ iff $[\![\phi]\!]^{s} = 0$.
b. $[\![\phi\rightarrow\psi]\!]^{s} = 1$ iff $[\![\phi]\!]^{s} = 0$ or $[\![\psi]\!]^{s} = 1$.
c. $[\![\Box\phi]\!]^{s} = 1$ iff for all s'∈S such that sRs': $[\![\phi]\!]^{s'} = 1$.
d. $[\![\Diamond\phi]\!]^{s} = 1$ iff for some s'∈S such that sRs': $[\![\phi]\!]^{s'} = 1$.

QUESTION 2: Consider the Model in (12), where the arrows encode the accessibility relation R. Decide which of the formulae on the right are true in s_1 and which are true in s_2. [From Gamut p. 27]

(12) $[\![\mathbf{p}]\!]^{s1} = 1$ $[\![\mathbf{p}]\!]^{s2} = 0$



a) $\Diamond\Box p$
b) $\Box p \rightarrow \Box\Box p$
c) $\neg\Box p$
d) $p \rightarrow \Box\Diamond p$

2. INTENSIONAL PREDICATE LOGIC (INTPRL).

- Domain of individuals, and names.
  We consider that each situation s has its own domain of individuals, D^s_e. Even two different worlds (i.e, very large situations) may have different domains of individuals, since, e.g., I, Maribel, may exist in one but fail to exist in the other.
  Hence, the denotation of a name is dependent on the evaluation s: $[\![\textbf{Maribel}]\!]^s$ = me iff I exist in s. We will, thus, assume that names express partial constant functions of type <s,e> (Cf. Kripke: rigid designators).

2.1. Syntax of Modal PrL (alethic).

- Primitive vocabulary:

(15) Lexical entries, with a denotation of their own:
  a. A set of individual constants, represented with the letters **a**, **b**, **c**, **d**…
  b. A set of individual variables $\mathbf{x_0}$, $\mathbf{x_1}$, $\mathbf{x_2}$,… $\mathbf{y_0}$, $\mathbf{y_1}$, $\mathbf{y_2}$,… Individual constants and individual variables together constitute the set of terms.
  c. A set of predicates, each with a fixed n-arity, represented by **P**, **Q**, **R** …

(16) Symbols treated syncategorematically:
  a.The PL logical connectives.
  b. The quantifier symbols ∃ and ∀.
  c. The intensional (modal alethic) operators □ and ◊.

- Syntactic rules:

(17) a. If P is a n-ary predicate and t_1… t_n are all terms, then $P(t_1\ldots t_n)$ is an atomic formula.
  b. If ϕ is a formula, then $\neg\phi$ is a formula.
  c. If ϕ and ψ are formulae, then $(\phi\wedge\psi)$, $(\phi\vee\psi)$, $(\phi\rightarrow\psi)$, $(\phi\leftrightarrow\psi)$ are formulae too.
  d. If ϕ is a formula and v is a variable, then $\forall v\phi$, $\exists v\phi$ are formulae too.
  e. If ϕ is a formula, then $\Box\phi$ and $\Diamond\phi$ are formulae too.
  f. Nothing else is a formula in PrL.

2.2. Semantics of ModPrL (alethic).

- Model:

(18) A model for a ModPrL language consists of:
  a. a non empty set S of situations.
  b. a binary accessibility relation R in S.
  c. a domain of individuals for each situation s, D^s_e.
  d. a Lexicon assigning a denotation to every constant for each situation s.
  e. an assignment function g that assigns individuals to variables.

■ Semantic rules:

(19) a. If α is a constant (excluding syncategorematically treated symbols), then $[[\alpha]]^{s,g}$ is specified in the Lexikon for each s.

b. If α is a variable, then $[\![\alpha]\!]^{s,g} = g(\alpha)$

(20) a. If P is a n-ary predicate and $t_1 \ldots t_n$ are all terms, then, for any s,

$[\![P(t_1 \ldots t_n)]\!]^{s,g} = 1$ iff $[\![t_1]\!]^{s,g} \in D^s{}_e, \ldots, [\![t_n]\!]^{s,g} \in D^s{}_e$, and $<[\![t_1]\!]^{s,g}, \ldots, [[\, t_n]\!]^{s,g}> \in [\![P]\!]^{s,g}$

$[\![P(t_1 \ldots t_n)]\!]^{s,g} = 0$ iff $[\![t_1]\!]^{s,g} \in D^s{}_e, \ldots, [\![t_n]\!]^{s,g} \in D^s{}_e$, and $<[\![t_1]\!]^{s,g}, \ldots, [\![t_n]\!]^{s,g}> \notin [\![P]\!]^{s,g}$

If φ and ψ are formulae, then, for any situation s,

b. $[\![\neg\phi]\!]^{s,g} = 1$ iff $[\![\phi]\!]^{s,g} = 0$

$[\![\neg\phi]\!]^{s,g} = 0$ iff $[\![\phi]\!]^{s,g} = 1$

c. $[\![\phi\rightarrow\psi]\!]^{s,g} = 1$ iff $[\![\phi]]^{s,g} = 1$ and $[\![\psi]]^{s,g} = 1$
or $[\![\phi]\!]^{s,g} = 0$ and $[\![\psi]\!]^{s,g} = 1$
or $[\![\phi]\!]^{s,g} = 0$ and $[\![\psi]\!]^{s,g} = 0$

$[\![\phi\rightarrow\psi]\!]^{s,g} = 0$ iff $[\![\phi]\!]^{s,g} = 1$ and $[\![\psi]\!]^{s,g} = 0$.

d. $[\![\Box\phi]\!]^{s} = 1$ iff for all s'$\in$S such that sRs': $[\![\phi]]^{s'} = 1$

$[\![\Box\phi]\!]^{s} = 0$ iff there is an s'$\in$S such that sRs': $[\![\phi]\!]^{s'} = 0$

e. $[\![\Diamond\phi]\!]^{s} = 1$ iff there is an s'$\in$S such that sRs': $[\![\phi]\!]^{s'} = 1$

$[\![\Diamond\phi]\!]^{s} = 0$ iff for all s'$\in$S such that sRs': $[\![\phi]\!]^{s'} = 0$

f. If ϕ is a formula and v is a variable, then, for any situation s,

$[\![\forall v\phi]\!]^{s,g} = 1$ iff $[\![\phi]\!]^{s,gd/v} = 1$ for all the d $\in D^s{}_e$.

$[\![\forall v\phi]\!]^{s,g} = 0$ iff $[\![\phi]\!]^{s,gd/v} = 0$ for some d $\in D^s{}_e$.

g. If φ is a formula and v is a variable, then, for any situation s,

$[\![\exists v\phi]\!]^{s,g} = 1$ iff $[\![\phi]\!]^{s,gd/v} = 1$ for some d $\in D^s{}_e$

$[\![\exists v\phi]\!]^{s,g} = 0$ iff $[\![\phi]\!]^{s,gd/v} = 0$ for all d $\in D^s{}_e$

QUESTION 3: Is (21) a valid formula according to our semantics?

(21) $\Box(\phi\rightarrow\phi)$

3. NATURAL LANGUAGE AND INTENSIONALITY.

■ An intensional semantics for NatLg adds two characteristics to our current extensional framework:

- It adds some intensional operators, as we did with $\Box$ and $\Diamond$ in ModPL and ModPrL, except that NatLg is richer and combines modalities from different Intensional PLs and PrLs: epistemic, deontic, doxastic, bouletic, etc. We will specify the kind of modality (or Modal Base) as in (22):

(22) For any situations s and s', and for any accessibility relation R:

a. Epistemic R:
$s' \in Epi_x(s)$ iff s' conforms to what x knows in s.

b. Deontic R:
$s' \in Deo(s)$ iff all the obligations/requirements (to reach a given goal) are fulfilled in s', and s' is maximally similar to s otherwise.

c. Doxastic R:
$s' \in Dox_x(s)$ iff s' conforms to what x believes in s to be the case.

d. Bouletic:
$s' \in Bou_x(s)$ iff s' conforms to what x desires in s for it to be the case.

- It takes situations as part of the semantic values of the expressions. We will go only as far as implementing Intensional Type Theory for NatLg (expressions may have types $<s, \tau>$, though s is not a possible type by itself). The ultimate goal would be Two-Sorted Type Theory (where s is a basic type of its own).

3.1. Syntax.

(23) Vocabulary:

a. Constants of type <s,e>: **Mary, Ann, Tahiti**, …

b. Variables of type <s,e> (indices on pronouns and traces): **she_1, she_2, he_1, it_{24}, t_5, t_1**, …

c. Functional constants of different types (each type corresponding to one or more syntactic categories):

<s,<e,t>>	**run**, **cat**, …
<s, <e,<e,t>>>	**kiss, mother-of**, …
<s,<<s,et>, <s,et>>>	**alleged, counterfeit**, …
<s,<st, <st,t>>>	**if, if-and-only-if, and, or**
<s,<<s,et>,e>>	**the**
<s, <s,et>, <<s,et>, t>>	**every, some, most,…**
etc.	

d. Syncategorematically treated expressions: index n of movement on **$that/which_n$**.

(24) Syntactic Rules: GB and/or Minimalism (but adaptable to others).

3.2. Semantics.

■ Terms:

(25) Variable assignment: (possibly partial) function g: $|N \rightarrow D_e$ / $U\{D_a$: a is a type$\}$

(26) Lexical Entries:
a. If α is a constant, then $[\![\alpha]\!]^g$ is specified in the Lexikon for each s.
b. If α is index i on a trace or a pronoun (i.e. if a is α variable), then $[\![\alpha]\!]^g = \lambda s_s.\ g(i)$

(27) Some examples:

a. $[\![\textbf{she}_4]\!]^g$	=	$\lambda s_s.\ g(4)$
b. $[\![\textbf{Carmen}]\!]^g$	=	$\lambda s_s.\ c$
c. $[\![\textbf{cat}]\!]^g$	=	$\lambda s_s.\lambda x_e.\ CAT(x,s)$
d. $[\![\textbf{kiss}]\!]^g$	=	$\lambda s_s.\lambda x_e.\lambda y_e.\ KISS(y,x,s)$
e. $[\![\textbf{and}_{\textbf{IP}}]\!]^g$	=	$\lambda s_s.\lambda p_{<st>}.\lambda q_{<st>}.\ p(s) \wedge q(s)$
f. $[\![\textbf{the}]\!]^g$	=	$\lambda s_s.\lambda P_{<s,et>}.\ \iota x\ (\ P(s)(x)\ )$
g. $[\![\textbf{every}]\!]^g$	=	$\lambda s_s.\lambda P_{<s,et>}.\lambda Q_{<s,et>}.\ \forall x\ [P(s)(x) \rightarrow Q(s)(x)]$

■ Semantic rules:

(29) Non-Branching Nodes:
If α has the form $\begin{array}{c}\alpha\\ |\\ \beta\end{array}$, then $[\![\alpha]\!]^g = [\![\beta]\!]^g$.

(29) Functional Application (FA):
If α has the form $\begin{array}{cc} \multicolumn{2}{c}{\alpha} \\ \beta & \gamma \\ <s, <\sigma,\tau> & <s,\sigma> \end{array}$, then $[\![\alpha]\!]^g = \lambda s_s.\ [\![\beta]\!]^g\ (s)\ ([\![\gamma]\!]^g(s))$

(30) Intensional Functional Application (IFA):
If α has the form $\begin{array}{cc} \multicolumn{2}{c}{\alpha} \\ \beta & \gamma \\ <s, <\sigma,\tau> & \sigma \end{array}$, then $[\![\alpha]\!]^g = \lambda s_s.\ [\![\beta]\!]^g\ (s)\ ([\![\gamma]\!]^g)$

(31) Predicate Modification (PM):
If α has the form $\begin{array}{cc} \multicolumn{2}{c}{\alpha} \\ \beta & \gamma \end{array}$, and β and γ are both in $D_{<s,et>}$,
then $[\![\alpha]\!]^g = \lambda s_s.\lambda x_e.\ [\![\beta]\!]^g(s)(x) \wedge [\![\gamma]\!]^g(s)(x)$

(32) Predicate Abstraction (PA):
If α has the form $\begin{array}{cc} \multicolumn{2}{c}{\alpha} \\ i & \gamma \end{array}$, where $i \in |N$ and the type of i's trace is $<s,\sigma>$,
then $[\![\alpha]\!]^g = \lambda s_s.\lambda x \in D_\sigma.\ [\![\gamma]\!]^{gx/i}\ (s)$

(33) For any expression α, $[\![\alpha]\!] = \alpha'$ iff, for all assignments g, $[\![\alpha]\!]^g = \alpha'$.

QUESTION 4: Do the semantic computation, step by step, of (34) and (35) in the new intensional framework.

(34) The president of the U.S. loves Michelle Obama.

(35) Someone (=some person) from NY has-won-the-lottery.

■ Modal auxiliaries as intensional operators:

(36) $[\![\mathbf{must_{Deo}}]\!]^g$ = $\lambda s_s.\lambda p_{<st>}.\ \forall s'\ [\ s' \in Deo(s)\ \rightarrow\ p(s')=1\ ]$

(37) $[\![\mathbf{can_{Deo}}]\!]^g$ = $\lambda s_s.\lambda p_{<st>}.\ \exists s'\ [\ s' \in Deo(s)\ \wedge\ p(s')=1\ ]$

(38) $[\![\mathbf{must_{Epi}}]\!]^g$ = $\lambda s_s.\lambda p_{<st>}.\ \forall s'\ [\ s' \in Epi(s)\ \rightarrow\ p(s')=1\ ]$

(39) $[\![\mathbf{can_{Epi}}]\!]^g$ = $\lambda s_s.\lambda p_{<st>}.\ \exists s'\ [\ s' \in Epi(s)\ \wedge\ p(s')=1\ ]$

QUESTION 5: Assuming that Subjects are base-generated VP internally and that they can reconstruct to their base-generated position at LF, draw two LF syntactic structures for (40) that will capture the so-called de re / de dicto ambiguity [restrict yourself to deontic Modal Base here]. Do the same for (41) [use epistemic Modal Bease here]. Do both computations step by step.

(40) The president of the U.S. must love Michelle Obama.

(41) Someone (=some person) from NY must have-won-the-lottery.

■ Attitude verbs as intensional operators:

(40) $[\![\mathbf{believe}]\!]^g$ = $\lambda s_s.\lambda p_{<st>}.\lambda x_e.\ \forall s'\ [\ s' \in Dox_x(s)\ \rightarrow p(s')=1\ ]$

QUESTION 6: Propose a semantic value for (41) and complete the semantic computation of (42):

(41) $[\![\mathbf{hopes}]\!]^g$ =

(42) Anna hopes that every girl$_3$ introduced her$_3$ cat to Carmen.