QUESTION 2: We saw that **and** (and **or**) can conjoin two IPs, as in (20), and that they can conjoin two constituents of type <e,t>, as in (21). In fact, **and** and **or** are crosscategorial and can also conjoin two denotations of type <e,<e,t>>, two denotations of type <e,<e,<e,t>>>, two Determiner denotations, etc., as illustrated in (22)-(25). Spell out the lexical entries in (22b), (23b), (24b) and (25b). Then, conflate all the **and** denotations into one schema, and all the **or** denotations into another schema.

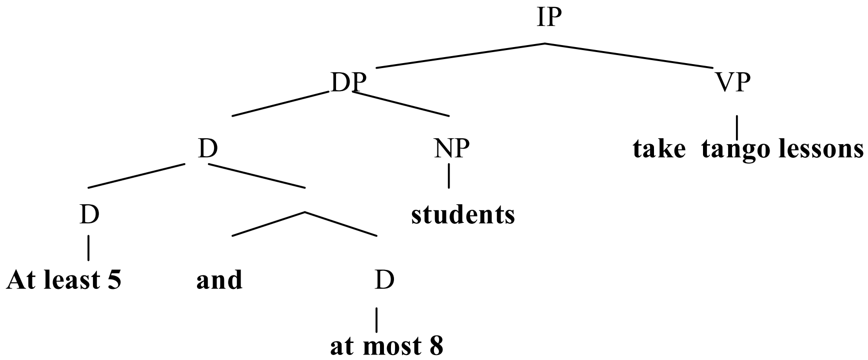
(20) a. It is raining and the sun is shining.
b. $[\![\textbf{and}_{IP}]\!]^{s,g}$ = $\lambda p_t.\lambda q_t.\ p=1 \wedge q=1$

(21) a. Ann is female and [from Tahiti].
b. $[\![\textbf{and}_{1\text{-place}}]\!]^{s,g}$ = $\lambda P_{<e,t>}.\lambda Q_{<e,t>}.\lambda x_e.\ P(x) \wedge Q(x)$

(22) a. Ann kissed and hugged Bill.
b. $[\![\textbf{and}_{Vtr}]\!]^{s,g}$ =

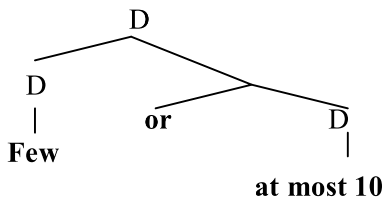
(23) a. Raquel showed and gave the book to Paul.
b. $[\![\textbf{and}_{Vditr}]\!]^{s,g}$ =

(24) a. At least 5 and at most 8 students take tango lessons.



b. $[\![\textbf{and}_{Det}]\!]^{s,g}$ =

(25) a. Few or at most 10 students take tango lessons.



b. $[\![\textbf{or}_{Det}]\!]^{s,g}$ =

3. Formal properties of Determiner denotations.

For any Determiner Denotation Det (hence, for any function of type $< <et>, <<et>t> >$), we can define the properties that follow. Take A, B to range over functions of type $<e,t>$, and take A' and B' to be the NatLg expressions denoting A and B.

(26) Det is REFLEXIVE iff
for all $A_{<e,t>}$ [such that Det(A)(A) is defined]: Det(A)(A) = 1

(27) Testing examples:
a. **Every** is reflexive, since, for an arbitrary A denoted by A', the following necessarily holds: ⟦ **Every A' (is) A'** ⟧ = 1
b. **At most n** is not reflexive, since, if $|A| \geq n+1$, the following does not hold:
⟦ **At most n A' (is/are) A'** ⟧ = 1

(28) Det is IRREFLEXIVE iff
for all $A_{<e,t>}$ [such that Det(A)(A) is defined]: Det(A)(A) = 0

(29) Testing examples:
a. **Not every** is irreflexive, since, for an arbitrary A (denoted by A'), the following necessarily holds: ⟦ **Not every A' (is) A'** ⟧ = 0
b. **At most n** is not irreflexive, since, if $|A| \leq n$, the following does not hold:
⟦ **At most n A' (is/are) A'** ⟧ = 0

(30) Det is SYMMETRIC iff
for all $A_{<e,t>}$ and $B_{<e,t>}$
[Det(A)(B) is defined iff Det(B)(A) is defined, and]
Det(A)(B) = 1 iff Det(B)(A) = 1

(31) Testing examples:
a. **Some** is symmetric, since, for an arbitrary A and B, the following holds:
⟦**Some A' (is/are) B'**⟧=1 iff ⟦**Some B' (is/are) A'**⟧=1
b. **Every** is not symmetric, since, for any two distinct A and B, it holds that:
if ⟦**Every A' (is) B'**⟧ =1 then ⟦**Every B' (is) A'**⟧ = 0

(32) Det is CONSERVATIVE iff
for all $A_{<e,t>}$ and $B_{<e,t>}$
[Det(A)(B) is defined iff Det(A)($\lambda x.A(x) \wedge B(x)$) is defined, and]
Det(A)(B) = 1 iff Det(A)($\lambda x.A(x) \wedge B(x)$) = 1
(In set notation, Det(A)(B) =1 iff Det(A)($A \cap B$) =1)

QUESTION 3: Test Determiners for conservativity. You can use the frame in (30).[1]
(33) ⟦___ **A' is B'**⟧=1 iff ⟦___ **A' is an A' that is B'**⟧=1

[1] Other important properties are extension, isomorphism and universality, discussed in: van der Does, J., and J. van Eijck. 1995. "Basic Quantifier Theory", in van der Does - van Eijck. 1995. *Quantifiers, Logic, and Language*. Stanford: CSLI Publications. Pp. 1-45. See also Keenan, E. L., and J. Stavi. 1986. "A Semantic Characterization of Natural Language Determiners", *Linguistics and Philosophy* 9: 253-326.

(34) Det is LEFT UPWARD MONOTONE iff
for all $A_{<e,t>}$, $B_{<e,t>}$ and $C_{<e,t>}$,
if $\forall x[A(x)\rightarrow B(x)]$ (in set terms, $A\subseteq B$),
$Det(A)(C) = 1$
[and Det(B)(C) is defined],
then $Det(B)(C) = 1$

(35) Sample test: **some** is left upward monotone.
$\forall x$ [⟦**cow**⟧ (x) → ⟦**mammal**⟧ (x)]
Some [cow] [just ate the grass in my garden] entails
Some [mammal] [just ate the grass in my garden].

(36) Sample test: **every** is not left upward monotone.
$\forall x$ [⟦**cow**⟧ (x) → ⟦**mammal**⟧ (x)]
Every [cow] [has four legs] does NOT entail
Every [mammal] [has four legs].

(37) Det is LEFT DOWNWARD MONOTONE iff
for all $A_{<e,t>}$, $B_{<e,t>}$ and $C_{<e,t>}$,
if $\forall x[A(x)\rightarrow B(x)$ (in set terms, $A\subseteq B$),
$Det(B)(C) = 1$
[and Det(A)(C) is defined],
then $Det(A)(C) = 1$

(38) Det is RIGHT UPWARD MONOTONE iff
for all $A_{<e,t>}$, $B_{<e,t>}$ and $C_{<e,t>}$,
if $\forall x[A(x)\rightarrow B(x)$ (in set terms, $A\subseteq B$),
$Det(C)(A) = 1$
[and Det(C)(B) is defined],
then $Det(C)(B) = 1$

(39) Det is RIGHT DOWNWARD MONOTONE iff
for all $A_{<e,t>}$, $B_{<e,t>}$ and $C_{<e,t>}$,
if $\forall x[A(x)\rightarrow B(x)$ (in set terms, $A\subseteq B$),
$Det(C)(B) = 1$
[and Det(C)(A) is defined],
then $Det(C)(A) = 1$

QUESTION 4: Evaluate the following determiners for monotonicity: **every**, **some**, **no**, **most**.

EXERCISE: Do exercises in book pp 152-3: **there**-insertion and negative polarity.

4. Presuppositional Determiners.

[See detailed discussion in Heim-Kratzer pp.153-172]

- Intuitions:

(40) a. The black cat snores.
Infelicitous utterance if the cardinality of the set contaning the relevant black cats is not equal to 1.

(41) a. Neither cat snores.
b. Both cats snore.
Infelicitous if the cardinality of the set of relevant cats is not equal to 2.

- Defining their denotations [Note that the presupposition intuition could not be captured if we take a Determiner to denote a *relation* (=set of pairs) between sets]:

(42) $[\![\textbf{Neither}]\!]^{s,g}$ =

(43) $[\![\textbf{Both}]\!]^{s,g}$ =

- More examples:

(44) The two cats snore.

- Controversial examples: according to Aristotelian logic, (45) entails (46). Note that the PrL rough translation (45') does not entail (46'). Strawson tries to capture the relation between (45) and (46) in NatLg by making **every** presupose that its first argument does not denote the empty set (or $\text{char}_{\varnothing}$).

(45) Every α is β.
(46) Some α is β.

(45') $\forall x\, [\, \alpha'(x) \rightarrow \beta'(x)\,]$
(46') $\exists x\, [\, \alpha'(x) \wedge \beta'(x)\,]$

(47) $[\![\textbf{every}]\!]^{s,\,g} = \lambda X_{<e,t>}{:}X\neq \text{char}_{\varnothing}\,.\, \lambda Y_{<e,t>}.\ \forall z\, (X(z) \rightarrow Y(z))$