POS, *-est* and reverse readings of *many* and *most* *

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1. Introduction

Since the seminal paper by Partee (1988), *many* and of its antonym *few* are known to give rise (at least) to two different readings: a cardinal reading, defined in (1-a), and a proportional reading, in (1-b). To see two examples, example (2-a) is true in scenario (2) in virtue of its cardinal reading and (2-b) in virtue of its proportional reading:[1]

(1) Many *P*s are *Q*.

a. CARDINAL reading: $|P \cap Q| > n$, where n is a large natural number.
b. PROPORTIONAL reading: $|P \cap Q| : |P| > p$, where p is a large proportion.

(2) Scenario: All the faculty children were at the 1980 picnic, but there were few faculty children back then. Almost all faculty children had a good time.

a. There were few faculty children at the 1980 picnic.
b. Many faculty children had a good time.

Additionally, Westerstahl (1985) noted a third reading for *many*: the so-called 'reverse' proportional reading. This is illustrated in (3)-(4). Besides its regular proportional reading —which is false in scenario (3), since, among all the Scandinavians, 14 does not count as many—, sentence (4) has a reading roughly paraphrasable as 'Many of the Nobel Prize winners are Scandinavians' that makes it true in that scenario (Westerståhl 1985).

(3) Scenario: Of a total of 81 Nobel Prize winners in literature, 14 are Scandinavians.

(4) Many Scandinavians have won the Nobel Prize in literature.

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[1] I will use P and Q interchangeable for natural language predicates and for their denotations.

A noteworthy characteristic of this third reading of *many* is that, contrary to the cardinal and proportional readings, the reverse proportional reading does not obey Conservativity, in (5). This goes contra one of the most well-known universals in semantic theory, according to which all natural language determiners are interpreted as conservative functions (Keenan & Stavi 1986; Barwise & Cooper 1981, U3; van der Does & van Eicjk 1996).

(5) A determiner denotation $f_{<et,<et,t>>}$ is conservative iff, for any P and $Q \in \mathrm{D}_{<e,t>}$:
$$f(P)(Q) = 1 \quad \text{iff} \quad f(P)(P \cap Q) = 1$$

A second notable property of the reverse proportional reading is that the determiner *most*, which is also typically analysed as expressing a proportion (= 'more than 1/2') does not allow for a parallel reverse reading (Westerståhl 1985, Cohen 2001): (6) does not have a reading paraphrasable as 'Most of the Nobel Prize winners are Scandinavians'.

(6) Most Scandinavians have won the Nobel Prize in literature.

The present paper argues that reverse readings occur not just with proportional *many* but also with cardinal *many* and to some extent with *most*, and that, once the source of "reversivity" is identified and factored out, all readings can be derived from determiner denotations that are obey Conservativity. More concretely, the paper takes as point of departure Romero's (2015) decomposition analysis of proportional *many* into the positive degree operator *POS* –responsible for whether we obtain a reverse or a non-reverse reading– and the parametrized determiner $\textsc{many}_{prop}$ –which is defined as a conservative function. The present paper extend this analysis to cardinal *many*, distinguishing between reverse and non-reverse cardinal readings, and to *most*, which in certain languages allows for the reverse cardinal reading.

The paper is organized as follows. We discuss the proper characterization of the reverse proportional reading in section 2. Section 3 provides some necessary background on *POS* and other degree operators with adjectives. Section 4 introduces Romero's (2015) decompositional analysis of the reverse proportional reading. In section 5, we extend this analysis to cardinal *many*. Section 6 extends it to *most*. Section 7 concludes.

2. Formal characterization of the reverse proportional reading

How can the reverse proportional reading exemplified in (3)-(4) be formally characterized in terms of truth conditions? We will review three (successive) characterizations in the literature. The first one follows Westerståhl's (1985) intuitive paraphrase and is given in (7) (see also Herburger (1997)). These truth conditions render $many_{rev.prop}$ non-conservative.

(7) Westerståhl (1985):

a. Paraphrase: 'Many of the Nobel Prize winners are Scandinavians.'

b. REVERSE PROPORTIONAL reading of *Many Ps are Q*:
$|P \cap Q| : |Q| > p$, where p is a large proportion.

Cohen (2001) points out a problem with this formalization: the truth conditions in (7-b) make no reference to the proportion $|P \cap Q| : |P|$, but this proportion matters. Consider (8). While three Andorrans having won the prize suffices to make sentence (8-a) true in scenario (8), it is doubtful that the same number renders sentence (8-b) true. Yet, the formalization in (7) only asks us to consider $|P \cap Q| : |Q|$, which is 3/112 for either sentence.

(8) Scenario: There are 112 Nobel Prize winners in literature. 3 out of a total of 60,000 Andorrans have won it. 3 out of a total of 20,000,000 Scandinavians have won it.

a. Many Andorrans have won the Nobel Prize in literature.
b. Many Scandinavians have won the Nobel Prize in literature.

The second formalization is due to Cohen (2001). He proposes that P behaves as a contrastive topic in the sentence and invokes a set of alternatives $\text{ALT}(P)$ (the set {Scandinavian, Mediterranean, Andorran,...} in our examples). This set of alternatives is then unionized to obtain $\cup\text{ALT}(P)$ (which amounts to the world population in our examples). The resulting paraphrase and truth conditions are in (9). They still render $many_{rev.prop}$ non-conservative.

(9) Cohen (2001):

a. Paraphrase: 'The proportion of Scandinavians that have won the Nobel Prize in literature is large compared to the proportion of the world population that have won the Nobel Prize in literature.'
b. REVERSE PROPORTIONAL reading of *Many Ps are Q*:
$|P \cap Q| : |P| > |\cup\text{ALT}(P) \cap Q| : |\cup\text{ALT}(P)|$

Romero (2015) notes that a problem arises with this formalisation as well: (9) makes no use of the point-wise alternatives $|P' \cap Q|{:}|P'|$, $|P'' \cap Q|{:}|P''|$, $|P''' \cap Q|{:}|P'''|$, etc., but these alternatives matter. To wit, consider the scenario variants in (10) and sentence (11):

(10) Scenario: There are 24 schools in this town, with 1000 students each. 140 out of the total 24000 students in this town got an A on the final exam. In the school we are referring to, 8 of the 1000 students got an A. The distribution of A-students in this town is such that, for most schools, ...

a. ...the number of A-students is 5 or 6, that is, the average bracket is [5,6].
b. ...the number of A-students is 6, 7, 8 or 9, that is, the average bracket is [6,7,8,9].

(11) Many students in this school got an A on the final exam.

In scenario (10-a), the distribution of A-students per school peeks at the interval [5, 6]. This makes 8 A-students count as many and sentence (11) is judged true. In scenario (10-b), the distribution of A-students peeks at the interval [6, 7, 8, 9]. This makes 8 A-students hardly count as many and thus sentence (11) is judged false. Yet, the analysis in (9) only asks us to consider the proportion of students of this school that got an A (namely, 8/1000 in

both scenarios) and the overall proportion of students in this town that got an A (namely, 140/24000 in both scenarios), hence wrongly predicting the same truth value in both cases.

The third an final characterization of the reverse proportional reading stems from Romero (2015). Crucially, the alternative proportions are taken into account and put together in a set, to which function θ applies to yield a threshold value such that any proportion greater than that threshold value counts as many. The resulting paraphrase and truth conditions are given in (12).[2] They still render $many_{rev.prop}$ non-conservative.

(12) a. Paraphrase: 'The proportion of Scandinavians that have won the Nobel Prize in literature is large compared to a threshold based on the proportions of inhabitants of other worlds regions that have won the Nobel Prize in literature.'

b. REVERSE PROPORTIONAL reading of *Many Ps are Q*:
$|P \cap Q|:|P| \quad > \quad \theta(\{|P' \cap Q|:|P'| : P' \in \text{ALT}(P)\})$

3. Background: Degree operators with adjectives

Adjectives may appear in the comparative, superlative and positive. Correspondingly, a family of degree operators *-er*, *-est* and *POS* has been defined. We focus on the latter two.

The denotation of the superlative morpheme *-est* is given in (13) and applied to an example in (14) (Heim 1999, a.o.). *-Est* asks for a comparison class $\boldsymbol{Q}$ and for a comparison term P. In our example, the comparison class corresponds to the set containing the set of degrees that each girl in Lucia's class (e.g., Greta, Sarah, Lucia, Liv, ...) reaches in tallness. Once this $\lambda\boldsymbol{Q}$-argument has been filled out (formally, via a context-dependent variable C), the degree phrase *[-est C]* constitutes a generalized quantifier over degrees, which QRs at LF to avoid type-mismatch, as in (14-b). The comparison term P is provided by the LF sister of *[-est C]*, yielding $\lambda d.\text{tall}(\text{lucia},d)$, i.e., the set of degrees that Lucia reaches in tallness. The sentence asserts that the degree set P corresponding to the comparison term is a proper superset of every degree set in $\boldsymbol{Q}$ that is different from P itself.[3]

(13) $[\![\text{-}est]\!] = \lambda\boldsymbol{Q}_{<dt,t>}.\lambda P_{<d,t>} : P \in \boldsymbol{Q} \,.\, \forall \boldsymbol{Q} \in Q[Q \neq P \rightarrow Q \subset P]$

(14) a. Lucía is tallest (among the girls in her class).

b. [[-est C] 1 [Lucia is t_1-tall]]

c. $\forall Q \in \{\lambda d.\text{tall}(\text{greta},d), \lambda d.\text{tall}(\text{sarah},d), \lambda d.\text{tall}(\text{lucia},d), \lambda d.\text{tall}(\text{liv},d), ...\}$
$[Q \neq \lambda d.\text{tall}(\text{lucia},d) \;\rightarrow\; Q \subset \lambda d.\text{tall}(\text{lucia},d)]$

[2] (12-b) is a simplification. If n-many alternatives to P end up yielding the same proportion, this proportion should be taken into account n-many times, not just once. There are (at least) two ways to ensure this: (i) the point-wise alternatives are put together in a sequence rather than a set, or (ii) we use an intensional version of θ that takes the intension rather than the extension of the natural languages predicates into consideration. Given that degree operators in general are sensitive to the predicate's intension (see footnote 3), I am inclined towards solution (ii).

[3] For simplicity, we treat degree operators extensionally. The intensional version of *-est* is (i):

(i) $[\![\text{-}est]\!] = \lambda\boldsymbol{Q}_{<<s,dt>,t>}.\, \lambda P_{<s,dt>} : P \in \boldsymbol{Q} \,.\, \lambda w.\, \forall Q \in \boldsymbol{Q}\, [Q \neq P \rightarrow Q(w) \subset P(w)]$

POS, -est *and reverse readings of* many *and* most

When the superlative morpheme *-est* is base-generated within a host NP, several readings are possible (Szabolcsi 1986, Heim 1999). Example (15) is ambiguous between the absolute reading paraphrased in (15-a) and the relative reading in (15-b). Furthermore, the exact relative reading depends (at least partly) on the information structure of the sentence. Under the relative reading, (16-a) with focus on *Mary* compares recipients of John's letters while (16-b) with focus on *John* compares senders of letters to Mary.

(15) John climbed the highest mountain.

a. Absolute: 'John climbed a mountain higher than any other mountain'.
b. Relative: 'John climbed a higher mountain than anybody else climbed'.

(16) a. John wrote the longest letter to $\text{Mary}_{\mathbf{F}}$.
b. $\text{John}_{\mathbf{F}}$ wrote the longest letter to Mary.

Here we are interested in the relative reading. Following (Heim 1999, a.o.), *-est* scopes out of its NP host, as in (17-a), the comparison term (17-b) is produced, and the comparison class *C* is retrieved (partly) from the focus value of the LF sister of *[-est C]* via the squiggle operator $\sim$, which requires (17-c). The resulting truth conditions are given in (17-d):

(17) Relative reading of *-est* in (15):

a. LF: [[-est C][1[$\text{John}_{\mathbf{F}}$ climbed A t_1-high mountain]]$\sim$C]
b. $[\![$*1[John climbed a t_1-high mountain]*$]\!] =$
λd.John climbed a d-high mountain
c. $[\![C]\!] \subseteq \{\lambda d'$.John climbed a d'-high mountain, $\lambda d'$.Bill climbed a d'-high mountain, $\lambda d'$.Paul climbed a d'-high mountain,$\ldots\}$
d. $[\![$(17-a)$]\!] = 1$ iff $\forall Q \in [\![C]\!]$ [$Q \neq \lambda d$.John climbed a d-high mountain $\rightarrow$
$Q \subset \lambda d$.John climbed a d-high mountain]

We turn now to the positive operator *POS*. It is defined in (18) and applied to an example in (19) (cf. Heim 2006, von Stechow 2009). Again, *POS* requests a comparison class argument $\boldsymbol{Q}$ —in our example, the set containing the set of degrees that each 8-year old reaches in tallness— and a comparison term argument P –the set of degrees that Lucia reaches in tallness. Function L in (18) takes a set of sets of degrees on a given scale and returns the so-called neutral segment on that scale (the segment including all degrees of height that count as neither tall nor short for an 8-year old) plus the previous and next points on that scale. As before, *[POS C]* QRs at LF: (19-b). Finally, the sentence asserts that the comparison term P is a superset of the middle interval selected by L, as in (19-c).

(18) $[\![POS]\!] = \lambda \boldsymbol{Q}_{<dt,t>}.\lambda P_{<d,t>} : P \in \boldsymbol{Q}.\ L_{<<dt,t>,<dt>>}(\boldsymbol{Q}) \subseteq P$

(19) a. Lucía is tall (for an 8-year old).
b. [[POS C] 1 [Lucia is t_1-tall]]
c. $L(\{\lambda d.\text{tall}(\text{bill},d), \lambda d.\text{tall}(\text{joe},d), \lambda d.\text{tall}(\text{lucia},d), \ldots\}) \subseteq \lambda d.\text{tall}(\text{lucia},d)$

Parallel to *-est*, the positive operator *POS* gives rise to an absolute/relative ambiguity, witness the readings in (20). Similarly to *-est*, the exact relative reading depends on what element *POS* associates with. If *POS*' associate in (21) is *Paul*, then the sentence refers to a hat that is expensive for somebody like Paul (e.g. unemployed people) to give; if the associate is *Mia*, then the sentence describes a hat that is expensive for somebody like Mia to get (e.g., for a 3-year old) (Schwarz 2010):

(20) Mia has an expensive hat.

a. Absolute: 'Mia has a hat that is expensive for a hat'
b. Relative: 'Mia has a hat that is expensive for somebody like Mia to have (e.g., for a 3-year old)'.

(21) Paul gave Mia an expensive hat.

Again, we will concentrate on the relative reading. Adapting Schwarz (2010)'s analysis to our lexical entry for *POS* [4] and leaving open whether *POS*' associate functions a Focus (F) or Contrastive Topic (CT), the derivation proceeds parallel to the one for *-est* above:

(22) Relative reading of *POS* in (20):

a. LF: [[POS C] [1 [Mia$_{\mathbf{F/CT}}$ has a t_1-expensive hat]] $\sim$ C]
b. $[\![$*1 [Mia$_{\mathbf{F/CT}}$ has a t_1-expensive hat]*$]\!]$ $=$ λd. Mia has a d-expensive hat
c. $[\![C]\!] \subseteq \{\lambda d'$. Mia has a d'-expensive hat, $\lambda d'$. Sue has a d'-expensive hat, $\ldots\}$
d. $[\![$(22-a)$]\!] = 1$ iff $L([\![C]\!]) \subseteq \lambda d$. Mia has a d-expensive hat

4. Analysis of the reverse proportional reading of *many* in Romero (2015)

Coming back to *many*, Romero's (2015) analysis of the reverse proportional reading holds the following tenets.

First, *many* is decomposed into the parametrized determiner MANY and the degree operator *POS* (cf. Penka (2011) on *few*). In other words, in the same way that the determiner *more* in (23-a) is in the comparative and can be decomposed into the parametrized determiner MANY and the comparative morpheme *-er* (Hackl 2000), *many* in (23-b) is in the positive and can be decomposed into MANY and the positive operator *POS*.

(23) a. Lucía has read more books (than Kira has).
b. Lucía has read many books (for an 8-year old).

[4] Instead of the the lexical entry for *POS* in (18), Schwarz (2010) uses (i), where (i) maps a scale $f_{<d,et>}$ to a property of individuals and $S(f)$ provides a standard degree for f.

(i) $[\![POS]\!] = \lambda f_{<d,et>}.\lambda x_e.\ \exists d[d > S(f) \wedge f(d)(x)]$

Second, Romero's (2015) lexical entry for proportional MANY is (24), which constitutes the parametrized determiner version of Partee's (1988) truth conditions in (1-b) (with the contextual variable p now replaced by a λ-bound variable d ranging over degrees). Crucially, there is only one proportional determiner MANY_{prop} —i.e., there is no reverse version $\text{MANY}_{rev.prop}$— and this unique MANY_{prop} is conservative.

(24) $[\![\text{MANY}_{prop}]\!] = \lambda d_d.\lambda P_{<e,t>}.\lambda Q_{<e,t>}.(|P \cap Q| : |P|) \geq d$

Third, *POS* with determiners does exactly what in does with adjectives. More concretely, just as in the relative reading of adjectives, *POS* in determiner *many* scopes sententially and retrieves a comparison class C from its syntactic scope based on its F-/CT-associate. And, just as with adjectives, the exact relative reading obtained depends on what element functions as *POS*' associate.

At this point, Romero (2015) presents a novel observation on *POS* with adjectives. In the relative readings considered in the literature, the associate of *POS* is external to the original host NP. In (21), for example, *Mia* or *Paul* is external to *[an expensive hat]*. Romero notes that *POS*' associate may be internal to the host NP as well. To see this, consider scenario (25) and sentence (26). The sentence has a reading that makes it true in this scenario, a reading roughly paraphrasable as 'Rockerfeller gave Kate a car and this present is inexpensive compared to his other presents to her'. Crucially, to obtain this reading via *POS*, we need a comparison class arising from *POS* having an associate internal to the host NP, namely, from *POS* having (at least) *car* as its associate, as shown in (27):

(25) Scenario: Rockefeller just gave Kate a very expensive car. Still, this present compares poorly to his previous astronomically expensive presents (e.g., an apartment in Manhattan, an island in the Pacific, etc.)

(26) (For what he has been giving her, now) Rockefeller gave Kate an inexpensive car.

(27) Rockefeller gave Kate an inexpensive $\text{car}_{\mathbf{F/CT}}$.

a. LF: [[POS C] [1 [Rockefeller gave Kate a t_1-inexpensive $\text{hat}_{\mathbf{F/CT}}$]] $\sim$ C]

b. $[\![$*[1 [Rockefeller gave Kate a t_1-inexpensive $\text{hat}_{\mathbf{F/CT}}$]]*$]\!]$ =
λd. R gave K an x that is a car and is d-inexpensive

c. $[\![C]\!] \subseteq \{\lambda d'$. R gave K an x that is a car and is d'-inexpensive,
$\lambda d'$. R gave K an x that is a Manhattan apartment and is d'-inexpensive,
$\lambda d'$. R gave K an x that is a Pacific island and is d'-inexpensive, $\ldots\}$

d. $[\![$(27-a)$]\!] = 1$ iff
$L([\![C]\!]) \subseteq \lambda d$. R gave K an x that is a car and is d-inexpensive

With the three ingredients above and with the innovation just described,[5] Romero's (2015) analysis of the proportional readings of *many* can be summarised as follows: The

[5]In section 6, we will see that *-est* in some languages can associate internally to the host NP as well. Note, furthermore, that example (26) is reminiscent of Heim & Kratzer's (1998) well-known case in (i)-(ii). To

regular proportional reading arises when *POS*' associate is external to the NP host and the reverse proportional reading obtains when the(/an) associate is internal to the NP host.

Let us derive these two readings for two examples, both using the conservative proportional determiner (24). We start with the regular/non-reverse proportional reading of (28) (=(2-b)). Here *POS* is associated with an element *external* to the host NP, e.g. with $good_{\mathbf{F/CT}}$ in (29-a), the comparison class C is restricted as in (29-b) and the resulting truth conditions are (29-c). This corresponds to Partee's (1988) regular proportional reading of *many*:

(28) Many faculty children had a $\text{good}_{\mathbf{F/CT}}$ time.

(29) Regular/non-reverse proportional reading of (28):

a. LF: [[POS C] [1[[t_1-MANY$_{prop}$ faculty children] has a $\text{good}_{\mathbf{F/CT}}$ time]] $\sim$ C]

b. $[\![C]\!] \subseteq \{\lambda d'.(|\{x : \text{fac-child}(x)\} \cap \{x : \text{have-good-time}(x)\}| : |\{x : \text{fac-child}(x)\}|) \geq d',$
$\lambda d'.(|\{x : \text{fac-child}(x)\} \cap \{x : \text{have-bad-time}(x)\}| : |\{x : \text{fac-child}(x)\}|) \geq d',$
$\lambda d'.(|\{x : \text{fac-child}(x)\} \cap \{x : \text{have-okey-time}(x)\}| : |\{x : \text{fac-child}(x)\}|) \geq d',$
$\ldots\}$

c. $L([\![C]\!]) \subseteq \lambda d.(|\{x : \text{fac-child}(x)\} \cap \{x : \text{have-good-time}(x)\}| : |\{x : \text{fac-child}(x)\}|) \geq d$

Now we turn to the reverse proportional reading in (30). Previous literature presented the intuition that the reverse proportional reading is available only if (part of) the N' complement of the determiner is focused (Herburger 1997) or functions as contrastive topic (Cohen 2001). Romero cashes out this observation in terms of association with *POS*. When *POS* is associated with a F/CT element *internal* to the host NP, namely $Scandinavians_{\mathbf{F/CT}}$ in (31-a), we obtain a comparison class C restricted as in (31-b):

(30) Many $\text{Scandinavians}_{\mathbf{F/CT}}$ have won the Nobel Prize in literature.

(31) Reverse proportional reading of (30):

a. LF: [[POS C] [1[[t_1-MANY$_{prop}$ $\text{Scandinavians}_{\mathbf{F/CT}}$] have won NP]] $\sim$ C]

b. $[\![C]\!] \subseteq$
$\{\lambda d'.(|\{x : \text{Scandinavian}(x)\} \cap \{x : \text{NP-winner}(x)\}| : |\{x : \text{Scandinavian}(x)\}|) \geq d',$
$\lambda d'.(|\{x : \text{Mediterranean}(x)\} \cap \{x : \text{NP-winner}(x)\}| : |\{x : \text{Mediterr.}(x)\}|) \geq d',$
$\lambda d'.(|\{x : \text{M.Eastern}(x)\} \cap \{x : \text{NP-winner}(x)\}| : |\{x : \text{M.Eastern}(x)\}|) \geq d', \ldots\}$

c. $L([\![C]\!]) \subseteq$
$\lambda d.(|\{x : \text{Scandinavian}(x)\} \cap \{x : \text{NP-winner}(x)\}| : |\{x : \text{Scandinavian}(x)\}|) \geq d$

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obtain the desired reading of (ii) via *POS*, we need the comparison class C in (iii), resulting from associating *POS* with the noun *elephant* –internally to the host NP– as well as with *Jumbo* –externally to the host NP:

(i) Scenario: To protect our city from the attack of King Kong, we have recruited elephant Jumbo, which is large for an elephant. As we see King Kong approaching and see his absolutely humongous size, we realise that Jumbo stands no chance, and say (ii).

(ii) Jumbo has no chance! He is just a small elephant.

(iii) $[\![C]\!] \subseteq \{\lambda d'.\text{Jumbo is an elephant and is } d'\text{-small}, \lambda d'.\text{King Kong is a monster and is } d'\text{-small}, \ldots\}$

The resulting truth conditions are (31-c). These correspond precisely to the truth conditions of the reverse proportional reading argued for in Section 2, repeated below as (32), except that now they are expressed in terms of degree sets rather than exact cardinalities. The comparison term gives us the set of real numbers $\lambda d.\ 0 < d \leq |P \cap Q|:|P|$ rather than the cardinality $|P \cap Q|:|P|$ in (32). For each alternative P' to the predicate *Scandinavians*, we have a parallel degree set $\lambda d.\ 0 < d \leq |P' \cap Q|:|P'|$ rather than the cardinality $|P' \cap Q|:|P'|$. By putting them together, we obtain the comparison class C in (31-b) rather than the set of cardinalities referred to in (32). Then L applies to C and yields a segment of the scale containing the neutral segment —which includes the threshold value that θ would yield for the corresponding comparison class of cardinalities— plus the previous and next points on the scale. The truth conditions in (31-c) state that the comparison term $\lambda d.\ 0 < d \leq |P \cap Q|:|P|$ is a superset of $L(C)$, which is true if and only if $|P \cap Q|:|P|$ is greater than the threshold value referred to in (32).

(32) REVERSE PROPORTIONAL reading of *Many Ps are Q*:
$|P \cap Q|:|P| \quad > \quad \theta(\{|P' \cap Q|:|P'| : P' \in \text{ALT}(P)\})$

To sum up, Romero's (2015) proposal derives the non-reverse and reverse proportional readings from a single, conservative lexical entry for the determiner MANY plus independently motivated association possibilities of the degree operator *POS*. What determines the type of proportional reading is what material *POS* associates with: if *POS*' associate is external to the host NP, the non-reverse proportional reading arises; if *POS* associates with an element internal to the host NP, the reverse proportional reading obtains.

5. Extension to cardinal *many*

In this section, we extend the analysis of the proportional readings of *many* in Romero (2015) to cardinal *many*. We start by decomposing cardinal *many* into the degree operator *POS* and the parametrized determiner MANY_{card} in (33). Parallel to proportional MANY_{prop}, our lexical entry for MANY_{card} constitutes the parametrized determiner version of Partee's (1988) truth conditions in (1-a):[6]

(33) $[\![\text{MANY}_{card}]\!] = \lambda d_d.\lambda P_{<e,t>}.\lambda Q_{<e,t>}.|P \cap Q| \geq d$

If *POS* in cardinal *many* has the same association possibilities as in adjectives and proportional *many*, *POS* should be able to have its associate external or internal to the host NP. We examine each possibility in turn.

When *POS* is associated with an element *external* to the host NP, the run-off-the-mill cardinal reading arises. This is exemplified in (34), where the subject *Lucía* functions as associate. The derivation is sketched in (35). The resulting truth conditions (35-c) state that

[6] I will not enter the discussion of whether cardinal *many* contains the parametrized determiner MANY_{card} in (33) or rather an adjectival version of it. See the arguments in Hackl (2000, pp. 99-102) for the determiner version and the advantages derived in Hackl (2009) from the adjectival version. If, in the end, an adjectival version is needed, the set of readings will be a superset of the set of readings to be described below.

the degrees d that Lucía has reached in reading a d-amount of books surpasses the average established for the comparison class C of 8-year olds, depicted in (35-b):

(34) (For an 8-year old,) Lucía$_{\mathbf{F/CT}}$ has read many books.

(35) a. LF: [[POS C] [1[Lucía$_{\mathbf{F/CT}}$ has read [t_1-MANY books]]] $\sim$ C]
b. $[\![C]\!] \subseteq \{\lambda d'.|\{x : \text{book}(x)\} \cap \{x : \text{read}(\text{lucia}, x)\}| \geq d',$
$\lambda d'.|\{x : \text{book}(x)\} \cap \{x : \text{read}(\text{anna}, x)\}| \geq d',$
$\lambda d'.|\{x : \text{book}(x)\} \cap \{x : \text{read}(\text{sarah}, x)\}| \geq d', \ldots\}$
c. $L([\![C]\!]) \subseteq \lambda d.|\{x : \text{book}(x)\} \cap \{x : \text{read}(\text{lucia}, x)\}| \geq d$

We argue that, additionally, *POS* in $many_{card}$ can associate with an element *internal* to the host NP. To see this, consider scenario (36) and example (37):

(36) Scenario: John is an avid reader and keeps all the books he reads in his huge library. But John dislikes Scottish authors and has read little from them. More concretely, there are five Scottish authors and, when looking at John's library, the speaker sees that John has read the following amounts of books by them:

McFire	McDawn	Hings	Keath	Douglas
1 (out of 2)	1 (out of 3)	1 (out of 5)	2 (out of 4)	6 (out of 60)

(37) (For how unappealing Scottish authors are to John,) John has read many books by Douglas$_{\mathbf{F/CT}}$.

Under the intended reading of (37), the comparison class does not include the degree sets d' that John and other people have reached in reading d'-many books by Douglas. Rather, the relevant comparison class consists of the degree sets d' that John has reached in reading d'-many books by Douglas and by other Scottish authors. That is, the F/CT-associate of *POS* is crucially internal to the host NP, as in the LF (38-a), and the comparison class has the shape in (38-b). The truth conditions (38-c) assert that the degrees d that John has reached in reading d-many books by Douglas surpass the average of degrees d' that John has reached in reading d'-many books by Scottish authors:

(38) a. LF: [[POS C] [1[John has read [t_1-MANY books by Douglas$_{\mathbf{F/CT}}$]]] $\sim$ C]
b. $[\![C]\!] \subseteq \{\lambda d'.|\{x : \text{book}(x) \wedge \text{by}(\text{mcfire}, x)\} \cap \{x : \text{read}(\text{john}, x)\}| \geq d',$
$\lambda d'.|\{x : \text{book}(x) \wedge \text{by}(\text{mcdawn}, x)\} \cap \{x : \text{read}(\text{john}, x)\}| \geq d',$
$\lambda d'.|\{x : \text{book}(x) \wedge \text{by}(\text{hings}, x)\} \cap \{x : \text{read}(\text{john}, x)\}| \geq d',$
$\lambda d'.|\{x : \text{book}(x) \wedge \text{by}(\text{keath}, x)\} \cap \{x : \text{read}(\text{john}, x)\}| \geq d',$
$\lambda d'.|\{x : \text{book}(x) \wedge \text{by}(\text{douglas}, x)\} \cap \{x : \text{read}(\text{john}, x)\}| \geq d'\}$
c. $L([\![C]\!]) \subseteq \lambda d.|\{x : \text{book}(x) \wedge \text{by}(\text{douglas}, x)\} \cap \{x : \text{read}(\text{john}, x)\}| \geq d$

Note, furthermore, that the intuitively detected reading is a cardinal reading, not a proportional reading. This can be seen if we consider the proportion of books by each Scottish

author read by John, listed in parentheses in scenario (36). Regardless of whether the speakers are aware of these proportions or not, the sentence is true because John having read 6 books by Douglas counts as many compared to the amounts —1, 1, 1, 2— of books by other Scottish authors read by John. If the detected reading were a proportional one, the sentence would be judged false, since 6 out of 60 books does not count as a large proportion compared to the proportions —1/2, 1/3, 1/5, 2/4— of books by other Scottish authors read by John. To bring up the parallelism between the reading at issue and the reverse proportional reading from Romero (2015), we will christen this reading 'reverse cardinal reading'.[7]

6. Extension to *most*

Just like *many* has been decomposed into the positive degree operator *POS* and MANY, it has been argued in the literature that *most* decomposes into the superlative operator *-est* and a MANY-part (cf. Hackl (2009). In this section, we extend the analysis of non-reverse/reverse cardinal/proportional readings of *many* to *most*.

In the analysis defended in the present paper, two independent factors play a role in determining what readings are available: **(i)** the choice between host-external vs. host-internal association of the degree operator *POS* or *-est*, which determines whether we obtain a non-reverse or a reverse reading, and **(ii)** the choice between MANY_{card} in (33) and MANY_{prop} in (24), which decides whether the reading is in essence cardinal or proportional. In the case of *many*, we have seen that all options are available in English, thus producing (at least) the four readings presented above. The question is what options are available for *most*, so that the reverse proportional reading of (6) is ruled out. Let us see each possibility in turn.

We start with non-reverse readings, produced by host-external association of the degree operator. That the grammar of English allows *-est* to have an associate external to the host NP has already been attested with adjectival examples in (15)-(17). This host-external association of *-est* is combined with MANY in example (39). (40) spells out the derivation using MANY_{card}, roughly following Hackl (2009),[8] and (41) using MANY_{prop}. Since the final truth conditions in (40-c) and (41-c) are logically equivalent, we cannot tell whether the MANY-part of *most* can only be MANY_{card}, only MANY_{prop} or may be both.

(39) $\text{John}_{\mathbf{F}}$ sent (the) most letters to Mary.

(40) a. LF: [[-est C] [1[$\text{John}_{\mathbf{F}}$ sent [t_1-MANY_{card} letters] to Mary]]$\sim$C]

b. $[\![C]\!] \subseteq \{\lambda d'.|\{x:\text{letter}(x)\} \cap \{x:\text{send}(\text{john},x,\text{mary})\}| \geq d',$
$\lambda d'.|\{x:\text{letter}(x)\} \cap \{x:\text{send}(\text{bill},x,\text{mary})\}| \geq d',$
$\lambda d'.|\{x:\text{letter}(x)\} \cap \{x:\text{send}(\text{paul},x,\text{mary})\}| \geq d', \ldots\}$

c. $[\![(39)]\!] = 1$ iff $\forall Q \in [\![C]\!]\ [Q \neq \lambda d.|\{x:\text{letter}(x)\} \cap \{x:\text{send}(\text{john},x,\text{m})\}| \geq d$
$\rightarrow Q \subset \lambda d.|\{x:\text{letter}(x)\} \cap \{x:\text{send}(\text{john},x,\text{m})\}| \geq d]$

[7] Doris Penka (p.c) has pointed out to me that this may in fact be the actual reading underlying some apparent cases of reverse proportional readings.

[8] Hackl (2009) uses an adjectival version of cardinal MANY instead (33). See footnote 6.

(41) a. LF: [[-est C] [1[John$_{\mathbf{F}}$ sent [t$_1$-MANY$_{prop}$ letters] to Mary]]~C]

b. $[\![C]\!] \subseteq \{\lambda d'.|\{x:\text{letter}(x)\} \cap \{x:\text{send}(\text{john},x,\text{mary})\}| : |\{x:\text{letter}(x)\}| \geq d',$
$\lambda d'.|\{x:\text{letter}(x)\} \cap \{x:\text{send}(\text{bill},x,\text{mary})\}| : |\{x:\text{letter}(x)\}| \geq d',$
$\lambda d'.|\{x:\text{letter}(x)\} \cap \{x:\text{send}(\text{paul},x,\text{m})\}| : |\{x:\text{letter}(x)\}| \geq d', \ldots\}$

c. $[\![(39)]\!] = 1$ iff
$\forall Q \in [\![C]\!]\ [Q \neq \lambda d.|\{x:\text{letter}(x)\} \cap \{x:\text{send}(\text{john},x,\text{m})\}| : |\{x:\text{letter}(x)\}| \geq d$
$\rightarrow Q \subset \lambda d.|\{x:\text{letter}(x)\} \cap \{x:\text{send}(\text{john},x,\text{m})\}| : |\{x:\text{letter}(x)\}| \geq d]$

We turn now to reverse readings. The first thing to check is whether the grammar of English allows for (sentence-scoping) *-est* to associate with an element internal to the host NP. We test this with adjectives in (42). The answer is 'no': (42) lacks the LF and reading in (42-a)-(42-b). But other languages, such as Bulgarian and Polish, do allow for host-internal association of *-est* (Pancheva & Tomaszewicz 2012, Tomaszewicz 2013): (43)-(44).

(42) John has the cheapest car$_{\mathbf{F}}$.

a. LF: [[-est C] [1[John has [A t_1-cheap car$_{\mathbf{F}}$]]] ~ C]

b. # 'John has a car that is cheaper than any other thing he has.'

(43) Ivan ima naj-dobri albumi na/to U2$_{\mathbf{F}}$. [Bulgarian]
Ivan has est-good albums of/by U2
'Ivan has better albums by U2 than by any other band.'

(44) Iwan ma naj-lepsze albumy U2$_{\mathbf{F}}$. [Polish]
Ivan has est-better albums U2
'Ivan has better albums by U2 than by any other band.'

Given that Bulgarian and Polish but not English seem to allow for host-internal association of *-est*, let us combine this association with MANY in these languages. This is done in (45) and (46). These sentences are translated as (47) by my informants and judged true in scenario (48). This shows that the sentences have the reverse cardinal reading in (49):

(45) Ivan ima naj-mnogo albumi na/to U2$_{\mathbf{F}}$. [Bulgarian]
Ivan has est-many albums of/by U2

(46) Ivan ma naj-wiecej albumów U2$_{\mathbf{F}}$. [Polish]
Ivan has est-many albums U2

(47) 'Ivan has more albums by U2 than by any other band.'

(48) Scenario: Ivan has 15 (out of the 45) albums released by U2, 8 (out of the 10) albums released by Frank Zappa and 5 (out of the 7) albums released by Prince.

(49) a. LF: [[-est C] [1[Ivan has [t_1-MANY$_{card}$ albums by U2$_{\mathbf{F/CT}}$]]] ~ C]

b. $[\![C]\!] \subseteq \{\lambda d'.|\{x :\text{album-by}(x,\text{U2})\} \cap \{x :\text{have}(\text{ivan},x)\}| \geq d',$
$\lambda d'.|\{x :\text{album-by}(x,\text{zappa}\} \cap \{x :\text{have}(\text{ivan},x)\}| \geq d',$
$\lambda d'.|\{x :\text{album-by}(x,\text{prince}\} \cap \{x :\text{have}(\text{ivan},x)\}| \geq d'\}$

c. $\forall Q \in [\![C]\!]\ [Q \neq \lambda d.|\{x :\text{album-by}(x,\text{U2}\} \cap \{x :\text{have}(\text{ivan},x)\}| \geq d \rightarrow$
$Q \subset \lambda d.|\{x :\text{album-by}(x,\text{U2}\} \cap \{x :\text{have}(\text{ivan},x)\}| \geq d]$

But, crucially, the Bulgarian and Polish sentences do not admit the paraphrase (50) and are consequently judged false in scenario (51). This shows that the sentences lack the reverse proportional reading in (52):[9]

(50) 'The proportion of albums by U2 that Ivan has is larger than the proportion he has for any other band.'

(51) Scenario: Ivan has 4 out of the 4 albums released by U2, 8 out of the 24 albums released by Frank Zappa and 5 out of the 50 albums released by Prince.

(52) a. LF: [[-est C] [1[Ivan has $[t_1$-MANY_{prop} albums by $\text{U2}_{\mathbf{F/CT}}]]] \sim$ C]

b. $[\![C]\!] \subseteq \{\lambda d'.(|\{x :\text{album}(x,\text{U2})\} \cap \{x :\text{have}(\text{ivan},x)\}| : |\{x :\text{album}(x,\text{U2})\}|) \geq d',$
$\lambda d'.(|\{x :\text{album}(x,\text{zappa})\} \cap \{x :\text{have}(\text{ivan},x)\}| : |\{x :\text{album}(x,\text{zappa})\}|) \geq d',$
$\lambda d'.(|\{x :\text{album}(x,\text{prince})\} \cap \{x :\text{have}(\text{ivan},x)\}| : |\{x :\text{album}(x,\text{prince})\}|) \geq d'\}$

c. $\forall Q \in [\![C]\!]$
$[\ Q \neq \lambda d.(|\{x :\text{album}(x,\text{U2})\} \cap \{x :\text{have}(\text{ivan},x)\}| : |\{x :\text{album}(x,\text{U2})\}|) \geq d \rightarrow$
$Q \subset \lambda d.(|\{x :\text{album}(x,\text{U2})\} \cap \{x :\text{have}(\text{ivan},x)\}| : |\{x :\text{album}(x,\text{U2})\}|) \geq d\ ]$

The picture that emerges is that the reverse proportional reading of *most* is in general missing not because of its reverse nature —which is blocked for *-est* in English but not in Bulgarian and Polish— but because of the proportional component. That is, though we cannot tell whether *most* is built from MANY_{card} or from MANY_{prop} with non-reverse readings, the present data on reverse readings suggest that it can only be built on MANY_{card}.

7. Conclusions

Westerståhl's (1985) reverse proportional reading of *many* has been analysed by Romero (2015) by decomposing *many* into a conservative determiner MANY, which obeys Conservativity, and the degree operator *POS*, which can associate with an element external or internal to the host NP, yielding the non-reserve and the reverse proportional readings respectively. Since the source of 'reversivity' is not exclusive to proportional *many*, this analysis predicts that reverse readings are more pervasive than originally thought. We have shown that this expectation is borne out: *many* displays also a reverse cardinal reading, and so does *most* in languages like Bulgarian and Polish.

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[9]Bulgarian and Polish 'many' allows for the reverse proportional reading in the Scandinavian sentence.

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