The conservativity of *many*[*]

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Abstract

Besides their conservative cardinal and proportional meanings, *many* and *few* have been argued to allow for a 'reverse' proportional reading that defies the convervativity universal (Westerståhl, 1985). We develop a compositional analysis that derives the correct truth conditions for this reading while maintaining conservativity. First, an amendment is proposed to Cohen's (2001) reverse proportional truth conditions. Second, mirroring the decomposition of other degree expressions like *tall*, *many* is decomposed into the parametrized determiner MANY and *POS*. *POS* is allowed to scope out of its host and scope sententially, and a comparison class C is retrieved via the (focus or contrastive topic) associate of *POS*. Keeping a unified conservative denotation for proportional MANY, the regular proportional reading obtains when *POS*' associate is external to the original host NP and the reverse proportional reading arises when it is internal to the host NP. The same applies to *few*.

1 Introduction

The study of generalized quantifiers has been a successful enterprise in semantic theory over several decades. One of its most important insights is that natural language determiners cannot denote just any function in $D_{<et,<et,t>>}$ but only those functions that satisfy certain constraints. Conservativity, defined in (1), is one of the constraints that have been argued for (Keenan & Stavi 1986; Barwise & Cooper 1981, U3; van der Does & van Eicjk 1996):

(1) A determiner denotation $f \in D_{<et,<et,t>>}$ is conservative iff, for any P and $Q \in \mathrm{D}_{<e,t>}$:
$f(P)(Q) = 1$ iff $f(P)(P \cap Q) = 1$

(2) Conservativity Universal:
Determiners in natural language are always interpreted as conservative functions.

An interesting case concerns the determiners *many* and *few*. Partee (1988) (and a long tradition thereafter) distinguishes two readings: the cardinal reading (3a)-(4a) and the proportional reading (3b)-(4b). To see these readings exemplified, consider sentences (6)-(7) in scenario (5). Sentence (6) is judged true in virtue of its cardinal reading and sentence (7) in virtue of its proportional reading. Once the context-dependent parameters n and p have been fixed for a given context, the functions denoted by $many_{\text{card/prop}}$ and $few_{\text{card/prop}}$ are conservative.

(3) Many Ps are Q.
- a. CARDINAL reading: $|P \cap Q| > n$, where n is a large natural number.
- b. PROPORTIONAL reading: $|P \cap Q| : |P| > p$, where p is a large proportion.

(4) Few Ps are Q.
- a. CARDINAL reading: $|P \cap Q| < n$, where n is a small natural number.

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b. PROPORTIONAL reading: $|P \cap Q| : |P| < p$, where p is a small proportion.

(5) Scenario: All the faculty children were at the 1980 picnic, but there were few faculty children back then. Almost all faculty children had a good time.

(6) There were few faculty children at the 1980 picnic.

(7) Many faculty children had a good time.

However, Westerståhl (1985) famously noted an additional reading of *many*, the so-called "reverse" proportional reading. Besides its regular proportional reading, which is false in scenario (8) (since, among all the Scandinavians, 14 does not count as many), sentence (9) has another proportional reading roughly paraphrasable as 'Many winners of the Nobel Prize in literature are Scandinavians' that makes it true in that scenario. The same point has been made for *few* (Herburger, 1997): Sentence (10) has a reading paraphrasable as 'Few applicants were cooks'. Formalizing these intuitive paraphrases gives us the truth conditions in (11)-(12). Crucially, these truth conditions render $many_{\text{rev-prop}}$ and $few_{\text{rev-prop}}$ non-conservative.

(8) Scenario: Of a total of 81 Nobel Prize winners in literature, 14 come from Scandinavia.

(9) Many Scandinavians have won the Nobel Prize in literature.

(10) Few cooks applied.

(11) Many Ps are Q.
REVERSE PROP. reading: $|P \cap Q| : |Q| > p$, where p is a large proportion.

(12) Few Ps are Q.
REVERSE PROP. reading: $|P \cap Q| : |Q| < p$, where p is a small proportion.

Efforts have been made in the literature to derive the reverse proportional reading of *many* and *few* in a principled way (Cohen, 2001; Herburger, 1997; de Hoop & Solà, 1996, a.o.), the key issue being whether, in such a principled derivation, the determiners remain conservative or challenge the conservativity universal.

The goal of this paper is two-fold: (i) to clarify the exact truth conditions of the reverse proportional reading and, (ii) building on Romero (2015), to derive these truth conditions compositionally while maintaining conservativity.

For the first goal, we will propose an amendment to Cohen's (2001) truth conditions.

For the second goal, the point of departure is the observation in the literature that the reverse proportional reading is available only if (part of) the N' complement of the determiner is focused (F) (Herburger, 1997) or functions as contrastive topic (CT) (Cohen, 2001), as indicated in (13)-(14). In a nutshell, our proposal will be the following. Just like degree adjectives like *tall* decompose into the stem TALL and the positive degree operator *POS* (Heim (2006); von Stechow (2009)), the determiners *many* and *few* decompose into MANY+*POS* and FEW+*POS* respectively. The determiners MANY and FEW will be defined as conservative. *POS* in determiners does exactly what it does in adjectives: it scopes out of its host and combines with the appropriate comparison class C via an associate, which we will implement as a F- or CT-associate.[1] Crucially, the F/CT associate of *POS* may lie outside its original host –as noted in the literature– or inside it –as we will observe in this paper. We will show that whether we obtain the regular or the reverse proportional reading of *many* / *few* depends on whether the F/CT-associate of *POS* is external or internal to the original host NP.

[1] I will talk about the F/CT associate of *POS* loosely, without commitment as to whether *POS* is conventionally or non-conventionally F- (or CT-) sensitive (see Beaver & Clark (2008)).

(13) Many Scandinavians$_{\mathbf{F/CT}}$ have won the Nobel Prize in literature.

(14) Few cooks$_{\mathbf{F/CT}}$ applied.

The rest of the paper is organized as follows. Section 2 takes a closer look at the truth conditions corresponding to the reverse proportional reading. Section 3 provides some background on *POS* with adjectives and presents the novel observation that *POS*'s associate can be internal to the original host NP. Section 4 spells out the proposal. Section 5 examines some further predictions. Section 6 concludes.

2 Truth conditions of the reverse proportional reading

We start with the truth conditions suggested by Westerståhl's (1985) intuitive paraphrase:

(15) Westerståhl (1985):

a. Paraphrase: 'Many of the Nobel Prize winners are Scandinavians.'

b. REVERSE PROPORTIONAL reading of *Many Ps are Q*:
$|P \cap Q| : |Q| > p$, where p is a large proportion.

Cohen (2001) shows that this characterization of the reading is incorrect: the truth conditions in (15b) make no reference to the proportion $|P \cap Q| : |P|$, but this proportion matters. To see this, consider scenario (16). While two Andorrans having won the prize suffices to make sentence (17) true, it is doubtful that the same number renders sentence (9) true. Yet, the formalization in (15b) only asks us to consider the proportion of winners that are Andorrans/Scandinavians (i.e., $|P \cap Q| : |Q|$), which is 2/112 for either sentence. Thus, for a contextually given value of p, (15b) wrongly predicts both sentences to be have the same truth value. What these examples show is that the proportion of Andorrans/Scandinavians that have won the prize (namely 2/60,000 vs. 2/20,000,000) plays a decisive role.

(16) Scenario: 112 Nobel Prize winners in literature. 2 out of a total of 60,000 Andorrans have won it. 2 out of a total of 20,000,000 Scandinavians have won it.

(17) Many ANDORRANS have won the Nobel Prize in literature.

To solve this problem, Cohen (2001) factors $|P \cap Q|{:}|P|$ into the truth conditions of *many*. Furthermore, P is argued to function as a contrastive topic and to invoke a set of alternatives ALT(P), to which $\cup$ is applied to yield $\cup$ALT(P). In our examples, ALT(P) is the set {Scandinavian, Mediterranean, Middle Eastern, Andorran, ...} and $\cup$ALT(P) is the set containing the world population. Cohen's intuitive paraphrase and proposed truth conditions are in (18). These truth conditions still render *many* non-conversative.

(18) Cohen (2001):

a. Paraphrase: 'The proportion of Scandinavians that have won the Nobel Prize in literature is large compared to the proportion of the world population that have won the Nobel Prize in literature.'

b. REVERSE PROPORTIONAL reading of *Many Ps are Q*:
$|P \cap Q| : |P| > |\cup \text{ALT}(P) \cap Q| : |\cup \text{ALT}(P)|$

We point out that Cohen's characterization of the reverse proportional reading is not fully correct either: (18) makes no use of the point-wise alternatives $|P' \cap Q|{:}|P'|$, $|P'' \cap Q|{:}|P''|$,

$|P''' \cap Q|{:}|P'''|$, etc., but these alternatives matter. To see this, consider sentence (19) on the two scenarios below. Under the regular proportional reading, the sentence is false in both scenarios (since, among the 1000 students in this school, 8 does not count as many). But, under the reverse proportional reading, the truth value intuitively differs in the two scenarios, even though the proportion of students of this school that got an A (namely, 8/1000) and the overall proportion of students in this town that got an A (namely, 140/24000) is the same in both scenarios. The truth values differ because the distribution of the alternative proportions matters. In scenario (20), the distribution of A-students per school peeks at the interval [5, 6]. This makes 8 A-students count as many and the sentence is judged true. In scenario (21), the distribution of A-students peeks at the interval [6, 7, 8, 9]. This makes 8 A-students hardly count as many and thus the sentence is intuitively judged false.

(19) Many students in this school got an A on the final exam.

(20) Scenario: 24 schools in this town, with 1000 students each. 140 out of the total 24000 students in this town got an A on the final exam. In the school we are referring to, 8 of the 1000 students got an A. For most schools, the number of students that got an A ranges between 5 and 6, e.g. as in Fig. 1.

(21) Scenario: 24 schools in this town, with 1000 students each. 140 out of the total 24000 students in this town got an A on the final exam. In the school we are referring to, 8 of the 1000 students got an A. For most schools, the number of students that got an A ranges between 6 and 9, e.g. as in Fig 2.

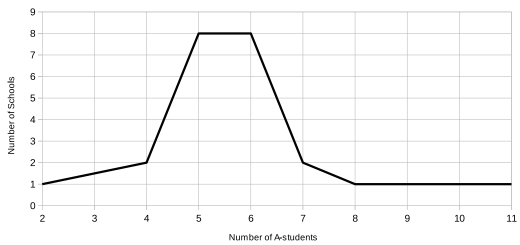

Figure 1

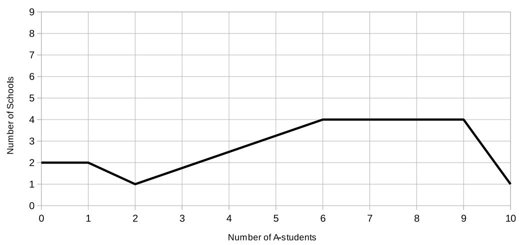

Figure 2

To solve this problem, the alternative proportions have to be factored in, as paraphrased and formalized in (22). The function θ combines with the set containing all these alternative proportions and yields a threshold value for that set, to which the original proportion $|P \cap Q| : |P|$ is compared.[2]

(22) a. Paraphrase: 'The proportion of Scandinavians that have won the Nobel Prize in literature is large compared to a threshold based on the proportions of inhabitants of other worlds regions that have won the Nobel Prize in literature.'

b. Reverse Proportional reading of *Many Ps are Q*:

$$|P \cap Q|{:}|P| \quad > \quad \theta(\{|P' \cap Q|{:}|P'| : P' \in \text{ALT}(P)\})$$

Note that these truth conditions still make reverse proportional *many* non-conservative. This takes us to our second goal: to arrive a these correct truth conditions compositionally while maintaing that all natural language determiners denote conservative functions.

[2]We leave open what mathematical operations θ applies to that set to obtain the threshold value. See Schöller & Franke (2015) for an algorithm compatible with (22), developing ideas from Fernando & Kamp (1996).

3 *POS* with adjectives

3.1 Background on *POS* with adjectives

Adjectives may appear in the comparative, superlative or positive form. This gives us the family of degree operators defined in (23)-(25), where $Q_{<d,t>}$ in *-er* corresponds to the second comparison term and $\boldsymbol{Q}_{<dt,t>}$ in *-est* and *POS* corresponds to the comparison class (Heim, 1999, 2006; von Stechow, 2009). For *POS*, L takes a set of sets of degrees on a given scale (e.g. the set containing, for each 8-year old x, the set of degrees of tallness x reaches) and returns the so-called neutral segment on that scale (the interval of degrees that make an 8-year old neither tall nor short plus the maximal degree lower than that interval and plus the minimum degree higher than that interval). This is depicted in (26).

(23) $[\![\textit{-er}]\!] = \lambda Q_{<d,t>}.\lambda P_{<d,t>}.Q \subset P$

(24) $[\![\textit{-est}]\!] = \lambda \boldsymbol{Q}_{<dt,t>}.\lambda P_{<d,t>}.\forall Q \in \boldsymbol{Q}[Q \neq P \rightarrow Q \subset P]$

(25) $[\![\textit{POS}]\!] = \lambda \boldsymbol{Q}_{<dt,t>}.\lambda P_{<d,t>}.L_{<<dt,t>,<dt>>}(\boldsymbol{Q}) \subseteq P]$

(26) |- - - - - - - - - - - - - - - - -[///////]- - - - - - - - - - - - - - - - ∞

Once the λQ-argument has been filled up (by the denotation of overt material or by a context-dependent variable C), we have a generalized quantifier over degrees, which must move out of its original host to gain scope, as in (27). The resulting truth conditions are in (28)-(30):[3]

(27) LF: [[-er/-est/POS C] 1 [Lucia is t_1-tall]]

(28) a. (Greta is 1,26m). Lucía is taller (than that).
b. $\lambda d.\text{tall}(\text{greta}, d) \subset \lambda d.\text{tall}(\text{lucia}, d)$

(29) a. Lucía is tallest (among the girls in her class).
b. $\forall Q \in \{\lambda d.\text{tall}(\text{greta}, d), \lambda d.\text{tall}(\text{sarah}, d), \lambda d.\text{tall}(\text{lucia}, d), \lambda d.\text{tall}(\text{liv}, d), \ldots\}$
$[Q \neq \lambda d.\text{tall}(\text{lucia}, d) \rightarrow Q \subset \lambda d.\text{tall}(\text{lucia}, d)]$

(30) a. Lucía is tall (for an 8-year old).
b. $L(\{\lambda d.\text{tall}(\text{valentin}, d), \lambda d.\text{tall}(\text{jonah}, d), \lambda d.\text{tall}(\text{lucia}, d), \ldots\}) \subseteq \lambda d.\text{tall}(\text{lucia}, d)$

It has been noted that the superlative morpheme *-est* with adjectives allows for an absolute and a relative reading, as in (31), and that the exact relative reading depends (at least partly) on the information structure of the sentence, as illustrated in (32) (Heim, 1999; Szabolcsi, 1986). Here we are interested in the relative reading. As sketched in (33), under this reading *-est* scopes out of its NP host and the comparison class C is retrieved (partly) from the focus value of the LF sister of *[-est C]* via the squiggle operator (Heim, 1999):

(31) John climbed the highest mountain.
a. Absolute: "John climbed a mountain higher than any other (relevant) mountain".
b. Relative: "John climbed a higher mountain than anybody else (relevant) climbed".

(32) a. John wrote the longest letter to $\text{Mary}_{\mathbf{F}}$. $\mapsto$ compares *recipients* of John's letters
b. $\text{John}_{\mathbf{F}}$ wrote the longest letter to Mary. $\mapsto$ compares *senders* of letters to Mary

[3] For simplicity we treat degree operators extensionality. The intensional treatment is illustrated in (i).
(i) $[\![\textit{-est}]\!] = \lambda \boldsymbol{Q}_{<<s,dt>,t>}.\lambda P_{<s,dt>}.\lambda w.\forall Q \in \boldsymbol{Q}[Q \neq P \rightarrow Q(w) \subset P(w)]$

(33) Relative reading of *-est*:

a. LF: [[**-est C**][1[John$_{\mathbf{F}}$ climbed A t_1-high mountain]] $\sim$ C]

b. $[\![$*1[John climbed a t_1-high mountain]*$]\!] = \lambda d'$.John climbed a d'-high mountain

c. $[\![C]\!] \subseteq \{\lambda d'$. John climbed a d'-high mountain, Bill climbed a d'-high mountain, $\lambda d'$. Paul climbed a d'-high mountain, $\ldots\}$

d. $[\![(31)]\!] = 1$ iff $\forall Q \in [\![C]\!]$ $[Q \neq \lambda d.\exists x[\text{climb}(j,x) \wedge \text{mount}(x) \wedge \text{high}(x,d)] \rightarrow Q \subset \lambda d.\exists x[\text{climb}(j,x) \wedge \text{mount}(x) \wedge \text{high}(x,d)]]$

A parallel absolute/relative ambiguity has been detected for *POS* with adjectives, where, again, the exact relative reading depends on what element *POS* associates with (Schwarz, 2010). This is shown in (34) and (35). Schwarz (2010) extends Heim's (1999) analysis of *-est* to *POS*. Again, here we are interested in the relative reading, adapted from Schwarz (2010) in (36):[4]

(34) Mia has an expensive hat.

a. Absolute: 'Mia has a hat that is expensive for a hat'

b. Relative: 'Mia has a hat that is expensive for somebody like Mia to have (e.g., for a 3-year old)'.

(35) Paul gave Mia an expensive hat.
$\mapsto$ a hat that is expensive for somebody like Paul (e.g. unemployed people) to give
$\mapsto$ a hat that is expensive for somebody like Mia (e.g. a 3-year old) to get

(36) Relative reading of *POS*:

a. LF: [[**POS C**] [1 [Mia$_{\mathbf{F/CT}}$ has a t_1-expensive hat]] $\sim$ C]

b. $[\![C]\!] \subseteq \{\lambda d'$. Mia has a d'-expensive hat, $\lambda d'$. Katie has a d'-expensive hat, $\ldots\}$

c. $[\![(34)]\!] = 1$ iff $L([\![C]\!]) \subseteq \lambda d.\exists x[\text{have}(m,x) \wedge \text{hat}(x) \wedge \text{expensive}(x,d)]$

3.2 A novel observation on *POS* with adjectives

In the relative readings in (35) above, the associate of *POS* (namely, *Mia* or *Paul*) is external to the original host NP *[an expensive hat]*. We note that the associate may be internal to the host NP as well: Sentence (37) has a reading that makes it true in scenario (38) for the comparison class (39). This comparison class corresponds to having *car* as the associate of *POS*.

(37) (For what he has been giving her, now) Rockefeller gave Kate an inexpensive car$_{\mathbf{F/CT}}$.

(38) Scenario: Rockefeller just gave Kate a very expensive car. Still, this present compares poorly to his previous astronomically expensive presents (e.g. apartment in Manhattan, island in Pacific, etc.)

(39) $[\![C]\!] \subseteq \{\lambda d'.R$ gave K a d'-inexpensive car, $\lambda d'.R$ gave K a d'-inexpensive apartment in Manhattan, $\lambda d'.R$ gave K a d'-inexpensive island in the Pacific, $\ldots\}$

In sum, we have seen that adjectives decompose into STEM+*-er*/*-est*/*POS* and we have witnessed how *POS* operates in the relevant readings: it scopes out of its host NP to gain sentential scope and it retrieves its comparison class C (partly) from the LF sister of *[POS C]* by cycling in different alternatives to *POS*' associate. Furthermore, this associate may be

[4] The use of focus/topic alternatives is not from Schwarz (2010). Schwarz uses a 3-place lexical entry for *POS* and thus does not need to generate alternatives from the information structure of the sentence. We assume the 2-place entry and need to generate alternatives somehow. To this end, we will assume that the associate of *POS* (e.g. *Paul* or *Mia* in (35)) bears focal stress or functions as contrastive topic.

external or internal to the original host NP. A similar decomposition has been proposed for certain determiners: *more* decomposes as MANY+-*er* (Hackl, 2000) and *most* as MANY+-*est* (Hackl, 2009). In the next section, we will propose that *many* decomposes as MANY+*POS* and we will parsimoniously assume that the behaviour of *POS* witnessed in adjectives is paralleled in determiners.

4 Proposal

The ingredients of the proposal are the following:

i. *Many* is decomposed into the parametrized determiner MANY and the degree operator *POS*. Similarly, *few* is decomposed into the parametrized determiner FEW and *POS*.[5]

ii. There is only one proportional determiner MANY_{prop} and only one proportional determiner FEW_{prop}, both of which are conservative.

iii. Just as we saw with adjectives, *POS* in determiners *many* and *few* scopes sententially and retrieves a comparison class C from its syntactic scope based on its F-/CT-associate. The exact reading obtained depends on the associate.

In the following, we show that the regular proportional reading arises when *POS*' associate is external to the NP host and the reverse proportional reading obtains when the associate is internal to the NP host.

4.1 Proportional readings of *many*

Once we severe *POS* from *many*, we are left with two determiner morphemes MANY: the cardinal one in (40) and the proportional one in (41)[6]. Since we are interested in the proportional reading in this talk, we will concentrate on (41).

(40) $[\![\text{MANY}_{\text{card}}]\!] = \lambda d_d.\lambda P_{<e,t>}.\lambda Q_{<e,t>}.|P \cap Q| \geq d$

(41) $[\![\text{MANY}_{\text{prop}}]\!] = \lambda d_d.\lambda P_{<e,t>}.\lambda Q_{<e,t>}.(|P \cap Q| : |P|) \geq d$

When we use $\text{MANY}_{\text{prop}}$ and *POS* is associated with an element <u>external</u> to the host NP, the regular proportional reading arises:

(42) Many (of the few) faculty children had a $\text{good}_{\mathbf{F/CT}}$ time.

(43) (Regular) proportional reading:

a. LF: [[**POS C**] [1[[t_1-$\text{MANY}_{\text{prop}}$ faculty children] has a $\text{good}_{\mathbf{F/CT}}$ time]] $\sim$ C]

b. $[\![C]\!] \subseteq \{\lambda d'.(|\{x : \text{fac-child}(x)\} \cap \{x : \text{have-good-time}(x)\}| : |\{x : \text{fac-child}(x)\}|) \geq d',$
$\lambda d'.(|\{x : \text{fac-child}(x)\} \cap \{x : \text{have-bad-time}(x)\}| : |\{x : \text{fac-child}(x)\}|) \geq d',$
$\lambda d'.(|\{x : \text{fac-child}(x)\} \cap \{x : \text{have-regular-time}(x)\}| : |\{x : \text{fac-child}(x)\}|) \geq d', \ldots\}$

c. $L([\![C]\!]) \subseteq \lambda d.(|\{x : \text{fac-child}(x)\} \cap \{x : \text{have-good-time}(x)\}| : |\{x : \text{fac-child}(x)\}|) \geq d$

[5] The parametrized determiner FEW will be further decomposed into LITTLE+MANY in Section 5 (Heim, 2006). For the examples in the present section, the simpler decomposition suffices.

[6] Adjectival uses of cardinal *many*/*few*, as exemplified in (i), suggest that $\text{MANY}_{\text{card}}$ / FEW_{card} may be adjectives rather than a determiners. See also Hackl (2009)'s analysis of the absolute reading of *most* based on an adjectival version of $\text{MANY}_{\text{card}}$. We leave a potential extension in this direction for future research.

(i) The many/few students of the University of Konstanz protested.

When we use $\textsc{Many}_{\text{prop}}$ but *POS* is associated with an element internal to the host NP, we obtain the reverse proportional reading. The truth conditions derived in (45b)-(45c) correspond precisely to the characterization of the reverse proportional reading argued for in Section 2.

(44) Many Scandinavians$_{\mathbf{F/CT}}$ have won the Nobel Prize in literature.

(45) Reverse proportional reading:

a. LF: [[**POS C**] [1[[t_1-$\textsc{many}_{\text{prop}}$ Scandinavians$_{\mathbf{F/CT}}$] have won NP]] $\sim$ C]

b. $[\![C]\!] \subseteq \{\lambda d'.(|\{x : \text{Scandinavian}(x)\} \cap \{x : \text{NP-winner}(x)\}| : |\{x : \text{Scandinavian}(x)\}|) \geq d', \lambda d'.(|\{x : \text{Mediterranean}(x)\} \cap \{x : \text{NP-winner}(x)\}| : |\{x : \text{Mediterr.}(x)\}|) \geq d', \lambda d'.(|\{x : \text{M.Eastern}(x)\} \cap \{x : \text{NP-winner}(x)\}| : |\{x : \text{M.Eastern}(x)\}|) \geq d', \ldots\}$

c. $L([\![C]\!]) \subseteq \lambda d.(|\{x : \text{Scandinavian}(x)\} \cap \{x : \text{NP-winner}(x)\}| : |\{x : \text{Scandinavian}(x)\}|) \geq d$

4.2 Proportional readings of *few*

Once we separate *POS* from *few*, we are left with the parametrized determiners $\textsc{few}_{\text{card}}$ and $\textsc{few}_{\text{prop}}$ below. Again, we will concentrate on the latter:

(46) $[\![\textsc{few}_{\text{card}}]\!] = \lambda d_d.\lambda P_{<e,t>}.\lambda Q_{<e,t>}.|P \cap Q| < d$ (TO BE REVISED)

(47) $[\![\textsc{few}_{\text{prop}}]\!] = \lambda d_d.\lambda P_{<e,t>}.\lambda Q_{<e,t>}.(|P \cap Q| : |P|) < d$ (TO BE REVISED)

When we use $\textsc{few}_{\text{prop}}$ and *POS* is associated with an element in the sentence external to the host NP, the regular proportional reading obtains:

(48) Few (of the many) demonstrators had a good$_{\mathbf{F/CT}}$ time.

(49) (Regular) proportional reading:

a. LF: [[**POS C**] [1[[t_1-$\textsc{few}_{\text{prop}}$ demonstrators] has a good$_{\mathbf{F/CT}}$ time]] $\sim$ C]

b. $[\![C]\!] \subseteq \{\lambda d'.(|\{x : \text{demonstr}(x)\} \cap \{x : \text{have-good-time}(x)\}| : |\{x : \text{demonstr}(x)\}|) < d', \lambda d'.(|\{x : \text{demonstr}(x)\} \cap \{x : \text{have-bad-time}(x)\}| : |\{x : \text{demonstr}(x)\}|) < d', \lambda d'.(|\{x : \text{demonstr}(x)\} \cap \{x : \text{have-regular-time}(x)\}| : |\{x : \text{demonstr}(x)\}|) < d', \ldots\}$

c. $L([\![C]\!]) \subseteq \lambda d.(|\{x : \text{demonstr}(x)\} \cap \{x : \text{have-good-time}(x)\}| : |\{x : \text{demonstr}(x)\}|) < d$

When we use $\textsc{few}_{\text{prop}}$ but *POS* is associated with an element in the sentence internal to the host NP, the reverse proportional reading results, with the truth conditions we argued for:

(50) Few cooks$_{\mathbf{F/CT}}$ applied.

(51) Reverse proportional reading:

a. LF: [[**POS C**] [1[[t_1-$\textsc{few}_{\text{prop}}$ cooks$_{\mathbf{F/CT}}$] applied]] $\sim$ C]

b. $[\![C]\!] \subseteq \{\lambda d'.(|\{x : \text{cooks}(x)\} \cap \{x : \text{apply}(x)\}| : |\{x : \text{cooks}(x)\}|) < d', \lambda d'.(|\{x : \text{someliers}(x)\} \cap \{x : \text{apply}(x)\}| : |\{x : \text{someliers}(x)\}|) < d', \lambda d'.(|\{x : \text{waiters}(x)\} \cap \{x : \text{apply}(x)\}| : |\{x : \text{waiters}(x)\}|) < d', \ldots\}$

c. $L([\![C]\!]) \subseteq \lambda d.(|\{x : \text{cooks}(x)\} \cap \{x : \text{apply}(x)\}| : |\{x : \text{cooks}(x)\}|) < d$

In sum, we have proposed a compositional analysis that derives the correct truth conditions for the reverse proportional reading of *many* and *few*, and this has been achieved using only conservative determiners —namely, $\textsc{many}_{\text{prop}}$ and $\textsc{few}_{\text{prop}}$— and exploiting independently motivated properties of *POS*.[7]

[7] When cardinal $\textsc{many}_{\text{card}}$ and $\textsc{few}_{\text{card}}$ are used, different readings are derived too depending on whether the associate is external or internal to the host NP. See Romero (2015) for details. The reason why no attention has been drawn towards the reverse cardinal reading is that it does not give the appearance of non-conservativity.

5 Further predictions

We have argued that *many* and *few* decompose into MANY+*POS* and FEW+*POS* respectively and that *POS* scopes out of its host NP to gain sentential scope. If the pair MANY/FEW behaves like other degree antonym pairs, e.g. *tall*/*short*, we expect FEW to further decompose into MANY plus the negative element LITTLE (cf. Heim, 2006), defined in (52):

(52) $[\![\text{LITTLE}]\!] \quad = \lambda d_d.\lambda D_{<d,t>}.D(d) = 0$

Furthermore, if *POS* behaves like other degree operators, we expect it to be able to take scope in its own clause or in a higher clause in the relevant configurations. Such an ambiguity is found e.g. for the comparative morpheme *-er* in example (53): *required* scoping over *-er* produces reading (54) and *-er* scoping over *required* produces reading (55) (Heim, 2006):

(53) (This draft is 15 pages.) The paper is required to be less long than that.
  a. 'Being under 15 pages is a necessity.'
  b. 'Being under 15 pages is a possibility.'

(54) a. LF: [require [$_{\text{DegP}}$ **-er** than 15pp] 1[[$_{\text{DegP}}$ t_1 little] 2 paper is long t_2]]
  b. $\lambda w.\ \forall w' \in \text{Acc}(w)\ [\{15\text{pp}\} \subset \lambda d.\neg\text{long}(\text{paper}, d, w')]$

(55) a. LF: [[$_{\text{DegP}}$ **-er** than 15pp] 1[[$_{\text{DegP}}$ t_1 little] 2 required paper is long t_2]]
  b. $\lambda w.\ \{15\text{pp}\} \ \subset\ \lambda d.\exists w' \in \text{Acc}(w)[\neg\text{long}(\text{paper}, d, w')]$

Hence, if the decomposition analysis of *many* and *few* is on the right track, the proposed analysis predicts more LFs to be possible than an analysis where all the meaning components are fused together and thus must scope together, e.g. an analysis where the truth conditions in (56) are packed in an undecomposable entry for *few*. The examples of FEW$_{\text{card}}$ below suggest that the additional power of the decomposition analysis is needed: while (57)-(58) lead to a reading where *POS* and LITTLE scope under *required*, as shown in (59), (60)-(61) prompt a reading where *POS* and LITTLE scope over *required*, as in (62).

(56) REVERSE PROPORTIONAL reading of *Few Ps are Q*:
$$|P \cap Q|{:}|P| \quad < \quad \theta(\{|P' \cap Q|{:}|P'| : P' \in \text{ALT}(P)\})$$

(57) Scenario: Our grad students are stressed out. According Dr. Smith's prescription, the amount of reading they do should be lower than that of a regular grad student (whatever that is). That is, our grad students reading few papers (for a grad student) is a necessity.

(58) Smith requires our grad students$_{\mathbf{F/CT}}$ to read few papers (for a grad student).

(59) a. LF: [S. requires [[**POS** C] 1 [[$_{\text{DegP}}$ t_1 LITTLE] 2 [the stud.$_{\mathbf{F/CT}}$ to read t_2-MANY papers]]]]
  b. $\lambda w.\ \forall w' \in \text{Acc}(w)[L([\![C]\!]) \subseteq \lambda d.|\{x : \text{papers}(x, w')\} \cap \{x : \text{read}(\text{john}, x, w')\}| < d]$

(60) Scenario: For all full professors except for Prof. Smith, the minimum requirement in their courses is for our grad students to read (any) 30 papers. For Prof. Smith, the minimum requirement in his courses is for our grad students to read (any) 10 papers. That is, in Prof. Smith's courses, our grad students reading few papers is a possibility.

(61) (For how much full professors tend to require from grad students in their courses,) Smith$_{\mathbf{F/CT}}$ requires our grad students to read few papers.

(62) a. LF: [[**POS** C] 1 [[$_{\text{DegP}}$ t_1 LITTLE] 2 [Smith$_{\mathbf{F/CT}}$ requires [the students to read t_2-MANY papers]]]]
  b. $\lambda w.\ L([\![C]\!]) \subseteq \lambda d.\exists w' \in \text{Acc}(w)[|\{x : \text{papers}(x, w')\} \cap \{x : \text{read}(\text{stud.}, x, w')\}| < d]$

6 Conclusions

By decomposing *many* into the positive degree operator *POS* and the parametrized determiner MANY, the so-called reverse proportional reading has been derived while appealing solely to independently motivated behavior of *POS* and while keeping a single, conservative lexical entry for $\text{MANY}_{\text{prop}}$. The same holds for *few*. Importantly, contrary to the analyses by Westerståhl (1985) and Cohen (2001), the proposed analysis derives the correct truth conditions for this reading. Furthermore, the proposed decomposition correctly predicts the existence of further scopal readings that are unexpected in non-decomposition analyses.

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