Introduction. Set Theory.

1. Frege's Compositionality Principle.

- One central assumption in current semantic theory is the Principle of Compositionality from Frege:

  (1) **PRINCIPLE OF COMPOSITIONALITY**: The meaning of a complex expression is determined by the meaning of its parts and the way those parts are combined.

- We need to find out:
  - The meaning of each sentence part: word or larger phrase.
  - The semantic contribution of the way the parts are combined.
  - The meaning corresponding to an entire sentence.

2. The meaning of a sentence: Truth-conditional semantics.

- Truth-conditional approach to the meaning of sentences:

  (3) To know the meaning of a sentence is to know under which conditions—more technically, in which worlds or sit—that sentence is true.

- Model:
  - Worlds \(W_1, W_2, W_3, W_4, \ldots W_{10}\): worlds where Maribel lives in Paradies.
  - Worlds \(W_{11}, W_{12}, W_{13}, W_{14}, \ldots W_{20}\): worlds where Maribel lives in Petershausen.

- Maribel lives in Paradies.

- Hence, a theory of meaning pairs sentences with truth-conditions:

  (6) For any world \(w\), the interpretation function \(\langle \rangle\) takes a linguistic expression as input and yields as output its meaning / denotation in the specific world \(w\).

- Digression: object language vs. metalanguage.
  - The language whose semantics we are studying—namely, English, represented in boldface—is our object-language. In order to talk about it, though, we have to use a language too, our metalanguage. Our metalanguage happens to be English—normal font—enhanced with some symbols.

3. The meaning of words and phrases.

- Some phrases and words can be used to stand for or denote a concrete individual in the world. Instead of using that word or phrase, you could simply point at the real object in the actual world. The following are some examples:

  (8) Proper names:
    - Maribel, Lucia, Konstanz, Bodensee, G300.

  (9) Noun phrases with demonstratives:
    - This table here, that window over there, these chairs, those pens.

  **QUESTION 1**: Can we give the same treatment to the definite Noun Phrases in (10)? Compare them with (8).

  (10) Definite Noun Phrases:
    - the tallest mountain in the Pyrenees
    - the president of the USA in 2010

  (11) a. \([\text{Lucia}]^{\circ} = \)
    
  b. \([\text{Lucia}]^{\ast7} = \)

  (12) a. \([\text{the president of the USA in 2010}]^{\circ} = \)
    
  b. \([\text{the president of the USA in 2010}]^{\ast7} = \)

- However, some other phrases and words do not stand for or denote a concrete object:

  (13) Non-referential Noun Phrases:
    - a. No student is sick.
    - b. Every woman, talked to the cat sitting on her lap.

  (14) Verbs and adjectives:
    - Run, see, put, red, tall, blond.
Current semantic theory proposes to treat meanings as set-theoretic objects. Some Noun Phrases stand for or denote concrete individuals in the world, but other phrases denote more abstract entities: a set of individuals, a set of pairs of individuals, a relation between sets of individuals, etc.

(15) \[ \text{blond} \] = \{ Karen, Al, Patrick \}

\[ \Rightarrow \text{Set Theory} \]

4. Set Theory

4.1 Sets.

(16) A set is a collection of objects, unordered.

(17) \{ a, b, c \}

\{ x: x \text{ snores} \}

\{ x: x \text{ is a multiple of } 3 \}

(18) An object \( a \) is an ELEMENT of a set \( A \) (\( a \in A \)) if that object is a member of the collection \( A \).

E.g.: \( a \in \{ a, b, c \} \)

\( 9 \in \{ x: x \text{ is a multiple of } 3 \} \)

(19) A set \( A \) is a SUBSET of a set \( B \) (\( A \subseteq B \)) if all the elements of \( A \) are also in \( B \).

(20) The INTERSECTION of two sets \( A \) and \( B \) (\( A \cap B \)) is the set containing all and only the objects that are elements of both \( A \) and \( B \).

(21) The UNION of two sets \( A \) and \( B \) (\( A \cup B \)) is the set containing all and only the objects that are elements of \( A \), of \( B \), or of both \( A \) and \( B \).

(22) The COMPLEMENT of a set \( A \) (\( \bar{A} \text{ or } A' \)) is the set containing all the individuals in the discourse except for the elements of \( A \).

(23) The DIFFERENCE \( A - B \) is the set resulting from subtracting from \( A \) all the elements in \( B \). \( A - B \) is equivalent to \( A \cap \bar{B} \).

(24) The POWER SET of a set \( A \) (\( \mathcal{P}(A) \)) is the set whose members are all the subsets of \( A \).

**QUESTION 2:** Partee et al., chapter 1, exercise 1 p. 23.

**QUESTION 3:** Given the sets under (25) and assuming that the universe of the discourse is \( \cup \{ A, B, C, D, E, F, G \} \), list the members of the following sets:

(25) \( A = \{ 1, 2, 3, 4 \} \)

\( E = \{ 1, 2, \{ a, 1 \} \} \)

\( B = \{ a, b, c, d, e, f \} \)

\( F = \{ 1, c, d \} \)

\( C = \{ 1, 2 \} \)

\( G = \{ d, e, 2, 3 \} \)

\( D = \{ 1, 3, 4, a, b \} \)

(26) a. \( A \cap D = \)

b. \( A \cap B = \)

c. \( F \cap C = \)

d. \( C - D = \)

e. \( C - F' = \)

f. \( E \cap C = \)

g. (\( C \cup D \)) - (\( C \cap D \)) =

h. \( G \cap F = \)
i. \( B \cup E = \)

j. \( (E \cup B) \cap D = \)

**QUESTION 4** (for home): Partee et al., chapter 1, exercises 2, 6 and 7 (pp. 23-25).

4.2 Relations.

**Ordered Pairs and Cartesian Product:**

(27) Ordered pair/n-tuple: a set with n-elements where order matters. E.g.: \(<a, b>\)

(28) Cartesian Product:

\( A \times B = \{ (x, y): x \in A \text{ and } y \in B \} \)

**Relations:**

(29) A relation is a set of ordered pairs (or, more generally, of ordered n-tuple).

More formally, a relation from \( A \) to \( B \) is a subset of \( A \times B \).

(30) Relation "to be fond of", relation "to kiss", relation "to be the left of", etc.

(31) a. Domain of \( R \): \( \text{a}: \text{there is some b such that } <\text{a}, \text{b}> \in R \)

b. Range of \( R \): \( \text{b}: \text{there is some a such that } <\text{a}, \text{b}> \in R \)

4.3 Functions.

(32) A relation \( R \) from \( A \) to \( B \) is a function from \( A \) to \( B \) (\( F: A \rightarrow B \)) if:

a. Every member of \( A \) appears at least once as first member of a pair (except for partial "functions").

b. Every member of \( A \) appears at most once as first member of a pair.

(33) The function "to have as mother", the function "to be the successor of", etc.

\( F_{\text{have as mother}}(\text{maribel}) = \text{ramona} \)

\( F_{\text{successor}}(3) = 4 \)