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\* I would like to thank Manfred Krifka and, in my former life, Ulrich Felgner and Frieder Haug for teaching me model theory. They are not to blame if they did not succeed.

## The Lower Part of Event Ontology

### 1 Conflicting applications of event ontology

In this paper, I will address an apparent conflict between two applications of event ontology for natural language semantics. The conflict arises within the lower part of event ontology, and consists, briefly, in the following: Scholars who aim at modelling tense and aspect, and in particular the distinction between telic and atelic predicates, commonly assume that certain properties of events are inherited by all their parts. These are called homogeneous properties. On the other hand, recent proposals to model negative polarity items have to assume that there is a level where the parts of events are so small that they can no longer reasonably inherit any property that can be denoted by a natural language predicate. In the following two sections, I shall recapitulate the respective positions and list some armchair assumptions about events that come along with either one. In section 1.3. I will name three possible ways out of the dilemma, two of which will be elaborated in this paper.

#### 1.1 Aspect

Krifka (1989), following earlier work by Link (1986), can be seen as the groundbreaking elementary proposal to model the distinction between telic and atelic predicates on the basis of events. It is assumed that sentence radicals denote properties of events  $P$ . If the property  $P$  is quantised, the sentence makes a telic statement. If the property  $P$  is homogeneous, then the sentence makes an atelic statement. The simplest linguistic correlate to this distinction is the test of whether the duration of the eventuality described will be specified with an *in*-PP (telic) or a *for*-PP (atelic sentence). The simplest definitions of “being quantised” and “being homogeneous” are given in (1) and (2). Further refinements were discussed in subsequent literature but are of no immediate concern here. I use  $\subset$  for the part-of relation in the domain of events.

- (1)  $QUANT(P) \Leftrightarrow \exists e( P(e) \ \& \ \forall e'( e' \subset e \rightarrow \neg P(e') ) )$   
 (2)  $HOM(P) \Leftrightarrow \forall e \forall e'( P(e) \ \& \ e' \subset e \rightarrow P(e') )$

It is easy to see that these definitions rest crucially on further assumptions about event ontology. In particular, for  $QUANT$  to be meaningful we need to ensure that the event ontology as such does not have an atomic level. Events are conceived of much like the set of time intervals on the rationals, which does not have an atomic level of smallest intervals. Indeed, at least nonstative events  $e$  have a running time  $\tau(e)$  which is an interval on the time line. The

following assumptions, hence, seem to be uncontroversial for this kind of theory, even if single authors do not care to list them all explicitly. Note that the symbol  $<$  stands for temporal precedence on the time line, as well as temporal precedence between two (temporally located) events. The relation  $\subset$  stands for the part-of relation between two events as well as the subset relation between time intervals. Clearly, if  $e \subset e'$  then  $\tau(e) \subset \tau(e')$ .

- (3) There is no lower boundary to events:  
 $\forall e \exists e' (e' < e)$
- (4) Boolean Structure: There is a summation operation  $\oplus$  defined on events that adds up adjacent events (incl. overlapping events) to larger events:  
 $\forall e \forall e' ( \neg \exists e^* ( \tau(e) < \tau(e^*) < \tau(e') \rightarrow \exists f ( e \oplus e' = f ) )$   
 (< on time intervals is the partial ordering defined as  $I < J$  iff  $\forall i \forall j (i \in I \ \& \ j \in J \rightarrow i < j)$ )
- (5) Betweenness: Between any two events, there is another one.  
 $\forall e \forall e' ( e' \subset e \rightarrow \exists e^* ( e' \subset e^* \subset e ) )$
- (6) Differences: If  $e'$  is part of  $e$ , then there are non-overlapping  $e''$ ,  $e'''$  that add up  $e'$  to  $e$ :  
 $\forall e \forall e' ( e' \subset e \rightarrow$   
 $[ \exists e'' ( e' \oplus e'' = e \wedge \neg \exists e^* ( e^* \subset e' \wedge e^* \subset e'' ) )$   
 $\vee \exists e'' e''' ( e' \oplus e'' \oplus e''' = e \wedge \neg \exists e^* ( e^* \subset e' \ \& \ e^* \subset e'' ) \wedge \neg \exists e^* ( e^* \subset e' \ \& \ e^* \subset e''' ) ) ]$

Atelic predicates are modelled as homogeneous predicates (Link 1986, Krifka 1989 and subsequent, Piñón 2000 and others). Link (1986) pointed out that, at least in the domain of real matter and things, these assumptions are in fact wrong, physically speaking. He notes that the matter *gold*, for instance, has an atomic level (namely, the level of single gold atoms) even though the natural language term '*gold*' behaves as if it denoted a homogeneous property. Link, as well as later authors, assumes that natural language ontology need not be isomorphic to a physically tenable model of the world, because the facts and phenomena in natural language have not been shaped by modern quantum physics but by folk views about nature and matter. Krifka (1989) makes similar remarks with respect to the lower end of event ontology. The general strategy hence seems to be, to use folk models of the world and time because we want to model the grammatical effects of folk notions about the world and time.

## 2.1 Negative Polarity Items

In a series of papers that go back to Fauconnier (1975), the behavior of negative polarity items is modeled based on the fact that they describe the weakest possible case of a set of salient alternatives (Krifka 1995, Lahiri 1998, Eckardt

2003). A sentence like (7) is predicted to be well formed because *lift a finger* denotes the smallest possible way of lending help.

(7) *Tom did not (even) lift a finger in order to help me.*

Interestingly, many languages distinguish between so-called *weak* and *strong* negative polarity items where strong NPIs are virtually restricted to negated contexts and rhetorical questions. The NPI in (7) is commonly assumed to be of the strong type. Informants agree that downward entailing contexts like *few*, *rarely*, etc. do not license it (see (8)) and that a question like (9) can only be used as a rhetorical question.

(8) *\*Few people (even) lifted a finger in order to help me.*

(9) *Did Tom even lift a finger in order to help you?*

There is a general consensus on how the data in (7)-(9) should be derived from the lexical meaning of the NPI, which is that expressions like *lift a finger*, *bat an eyelash*, *drink a drop* etc. denote irrelevantly small events or objects. This general idea has received different analyses by various authors. Krifka (1998) models it in probabilistic terms and stipulates the following strict inequality of probabilities:

(10)  $p(\bigwedge_i \text{Tom did not do } a_i \text{ in order to help me}) > p(\text{Tom did not lift a finger})$

where the conjunction  $\bigwedge_i$  ranges over all possible alternative ways in which Tom could have helped me.

The distinct distribution of strong NPIs is derived from this inequality. The details of the theory will not be discussed further in this paper. While this probabilistic account for strong polarity sensitivity is logically consistent with other assumptions about event ontology, inequalities such as the one in (10) are hard to relate with intuitions about events and their sub-events. One may suspect that a full model-theoretic account for (10) needs to address similar issues like the ones treated in this paper.

In a related but different vein, van Rooy (2003) explains the rhetorical quality of the question in (9) essentially by remarking that a positive answer to this question would be downright absurd, given that the question, according to his background theory, is such that a positive answer would pragmatically implicate that „lifting a finger“ is also the maximal and hence the only thing that Tom undertook in order to help me. He states that this would not be a relevant act of helping, without further discussion. Eckardt (2004a: chap. 4, 2004b) takes up this position and elaborates the distinction between weak and strong negative polarity items on the basic assumption that weak NPIs denote small objects and events but ones that are still reasonable things to do. Strong NPIs, in contrast, denote eventualities that are so small that they no longer fall in the right kind of category. For example, *lifting one's finger* may be a

subevent of events of helping, but it is not an event of helping itself, and it can not occur in isolation (i.e. without an appropriate superevent of reasonable size). This relates to the observation that natural language terms like Dutch *ook maar* or German *auch nur*, if used in questions, lead to rhetorical questions that cannot possibly receive a positive answer.

The ontological implementation of the strong-weak distinction suggests the plausibility of axioms like the ones in (11) and (12). These hold in particular also for properties of events  $P$  that would be regarded as homogeneous in an aspectual theory.

(11) If you *really* look down into the lower end of ontology, some events  $\epsilon$  are just too small to count:

$$\forall e( P(e) \rightarrow \exists \epsilon ( \epsilon \subset e \wedge \neg P(\epsilon) ) )$$

(12) Can we tell where?

$$(a) \exists e( P(e) \wedge \forall e'( e' \subset e \rightarrow \neg P(e') ) ) \quad (\text{'yes'})$$

$$(b) \forall e( P(e) \rightarrow \exists e'( e' \subset e \wedge P(e') ) ) \quad (\text{'no'})$$

It is evident that assumptions like (11) are fatal for a predicate that an account of aspect would predict to be homogeneous. On the other hand, we might claim that negative polarity items which denote “minimal objects or events“ do exactly what one should not do according to the general guidelines of aspect theory, which is to zoom into the lower part of event ontology which is simplified and idealized in this kind of modelling.

Note that the present conflict is not an easy one between folk theory and physical theories about the world. Both ways to view the lower end of event ontology are supported by *linguistic* facts. Hence, there seem to be two different folk theories about very small events. What kind of viewpoint shift is occurring here?

As the summary above already suggests, several analyses of “ $P$ -events too poor to mention” can be imagined. For present purposes, I will hypothetically adopt the

- Strong position: There are events  $e$  below  $P$ -events that are not themselves in  $P$ —even if  $P$  is intuitively a homogeneous predicate (for instance ‘*walk a single step*’ is not something that is an event of *walking*.)

I will *not* defend the strong position as the best, or only possible one. The aim of this paper is to demonstrate how this position can be carried out. Before we turn to the details, I will lay out the roadmap of the paper in the following section.

### 1.3 Possible Solutions

What kind of “blindness” makes speakers prefer one kind of expectations on one occasion and another on another occasion? What kind of change in our world view takes place once we zoom in the lower end of ontology? Somewhat surprisingly, there are even two consistent answers to this question. The first one elaborates the idea that we make bold universal statements about events (like HOM) because *we ignore some events*. If we really take all events into account, we are forced to retract these strong universal statements. The step between view one and view two hence consists in increasing or reducing the underlying domain of events. From a superficial view, so to speak, we can not see all events and hence feel inclined to universal statements like HOM(P). The surprising part of this idea is that we seem to see a great many small events even before we took that closer look. How could we have overlooked so many of them? In section 2, I offer an application of a model theoretic construction to the domain of events which shows that this is logically possible. In section 3, I compute an actual example that might be useful as an illustration, or for concrete applications.

In section four, I turn to a second kind of explanation which rests on the assumption that we face an instance of the Sorites paradox. This view comes down to the claim that we make bold universal statements because we idealizingly *assume wrong properties* for some minor events. I will discuss one spellout of this view and turn to a final comparison in the last section.

## 2 Infinitesimal Events

Let  $L$  be a first order language that contains relations and functions appropriate to event ontology. Specifically, I will use a sortal distinction between events and time intervals (along with the classical sorts for individuals; I will ignore extensions to higher order logic in the sequel). The unary function symbol  $\tau$  will be interpreted as the function that maps each event onto its running time. The binary relation  $\subset$  will be defined both on the set of events as well as the set of time intervals. The binary relation  $\leq$  is defined primarily as the earlier-than relation on the set of time intervals. It can be shifted to the domain of events by assuming that  $e_1 \leq e_2$  iff  $\tau(e_1) \leq \tau(e_2)$ . Finally, the binary function  $\oplus$  is interpreted as event summation. For the present purposes I will assume that summation is restricted to temporally adjacent events. Nothing depends crucially on this assumption, but it is in the spirit of the general enterprise to see how two perfectly natural but contradictory views of event ontology relate to each other.

Let  $\mathcal{E} = (E, \tau, \oplus, <, \subset)$  be an event structure for such a first order language, and one that specifically verifies the  $L$ -axioms (3) to (6) above. We assume

moreover that there is at least one homogeneous predicate  $P$  that lives on  $\mathcal{E}$ . The model theoretic construction will be spelled out with reference to  $P$ . It can easily be modified so as to extend to further homogeneous predicates.

Definition: Let  $(e_i)_{i \in \mathbb{N}}$  be a sequence of events in  $\mathcal{E}$ . We call  $(e_i)_{i \in \mathbb{N}}$  zero-convergent iff

- $\forall i \forall j (i < j \rightarrow e_j \subset e_i)$
- $\neg \exists e \forall i (e \subset e_i)$

Let  $\Phi(\mathcal{E})$  be the set of all zero-convergent sequences in  $\mathcal{E}$ .

Definition: Let  $\approx$  be a relation on  $\Phi(\mathcal{E})$  that is defined as follows:

$$\begin{aligned} (e_i)_{i \in \mathbb{N}} \approx (f_i)_{i \in \mathbb{N}} \text{ iff} \\ \forall e_i \exists f_j (f_j \subset e_i) \\ \forall f_k \exists e_r (e_r \subset f_k) \end{aligned}$$

As a consequence,  $\approx$  is an equivalence relation on  $\Phi(\mathcal{E})$ .

Next, we will augment  $L$  to a richer language  $L(\mathcal{E})$  by adding constant names for all events in  $\mathcal{E}$ . Formally, we could do so by taking the respective domain of  $\mathcal{E}$ , indexing all its elements  $e$ , for example, as  $e$ , in order to avoid confusion between objects and language, and add these indexed elements as new constant symbols to  $L$ .

Now, consider the following sets  $\Psi$  of sentences in  $L(\mathcal{E})$ :

$$(14) \quad \Phi[(e_i)_{i \in \mathbb{N}}] := \{ x \subset e_i \mid i \in \mathbb{N} \} \cup \{ \neg \forall e (x \subset e) \}$$

An object that would make all statements in  $\Phi[(e_i)_{i \in \mathbb{N}}]$  true would be part of all events in  $(e_i)_{i \in \mathbb{N}}$  without being the zero event. The next goal we need to achieve is

- 1) to construct an event structure  $\hat{\mathcal{E}}$
- 2) that extends the original event structure  $\mathcal{E}$  and
- 3) contains new elements  $\varepsilon$  such that
- 4) for each one of the sets  $\Psi$  of sentences as in (14), there is some  $\varepsilon$  for which all the formulae in  $\Psi$  hold true at once.

The construction we are aiming for should add such very small elements for all zero-convergent sequences in  $\mathcal{E}$ . However, we also must account for cases in

which two such sequences converge to “the same point”. In order to avoid contradictions, we need to ensure that only one infinitesimal element will be added in such cases. Therefore, we will first identify all co-convergent sequences.

First, we will choose some representative sequence  $(e_i)_{i \in \mathbb{N}}$  for each of the equivalence classes modulo  $\approx$  in  $\Phi(\mathcal{E})$ . Remember that this sequence now stands as the representative for all further sequences that consist of different events but eventually dove-tail with this representative in such a way as to converge to the same (so far: abstract) mini-event. For each one of these representatives, we now take the respective set of formulae  $\Phi[(e_i)_{i \in \mathbb{N}}]$  as in (14) and keep it in stock. We need to keep the free variables in each of the  $\Phi[(e_i)_{i \in \mathbb{N}}]$  distinct. For the purpose of exposition here, I will use different letters  $x, y, z$  for different sets of formulae  $\Phi[(e_i)_{i \in \mathbb{N}}]$ ,  $\Phi[(f_i)_{i \in \mathbb{N}}]$ . Generally, we must use variables in the definition of  $\Phi[(e_i)_{i \in \mathbb{N}}]$  that are indexed with the respective sequence. I will *not* carry this out for obvious typographical reasons. Formally:

*Definition:* Choose a fixed set of representatives for the equivalence classes in

$\Phi(\mathcal{E})/\approx$ . Let

$$\Phi := \cup \{ \Phi[(e_i)_{i \in \mathbb{N}}] \mid (e_i) \text{ the representative of some equivalence class in } \Phi(\mathcal{E})/\approx \}$$

We now need to conjoin these formulae with the elementary theory of  $\mathcal{E}$ : Let therefore

$$\text{Th}(\mathcal{E}) := \{ \psi \mid \psi \text{ atomic sentence in } L(\mathcal{E}) \text{ and } \mathcal{E} \models \psi \}$$

Hence,  $\text{Th}(\mathcal{E})$  offers a full description of all elements in  $\mathcal{E}$ . Any model for  $\text{Th}(\mathcal{E})$  will therefore contain a substructure that is isomorphic to the original structure  $\mathcal{E}$ .

Finally, let us add the requirement that events between two P-events are again P-events:

$$\forall e \forall e' \forall e^* ( e' \subset e^* \subset e \ \& \ P(e) \ \& \ P(e') \rightarrow P(e^*) )$$

We can now turn to the construction of an event structure  $\hat{\mathcal{E}}$  which extends  $\mathcal{E}$  in a conservative manner, which contains infinitesimal events, and where these infinitesimal events are not in the extension of  $P$  even though they might be parts of larger events that are in the extension of  $P$ .

First observe that the following set of formulae is finitely consistent:

$$\Phi \cup \text{Th}(\mathcal{E}) \cup \{ \forall e \forall e' \forall e^* ( e' \subset e^* \subset e \ \& \ P(e) \ \& \ P(e') \rightarrow P(e^*) ) \}$$



If we take any finite subset  $\Delta$  of this set of formulae, we can prove its consistency by interpreting it in the old event structure  $\mathcal{E}$  from which we started. More specifically, there is an interpretation  $I$  of the constant symbols and a variable assignment  $g$  for the free variables in  $\Phi$  such that  $\mathcal{E} \models^{I,g} \delta$  for all formulae in  $\Delta$ . We can simply interpret all constant names  $\acute{e}$  in  $\Delta$  by the respective element  $e$  in  $\mathcal{E}$  and interpret the variables  $y, x, z$  as events that are part of all those larger events that are mentioned in  $\Delta$ , among the formulae collected in  $\Phi$ . Because there are only a finite number of such statements in  $\Delta$  and the decreasing sequences  $(e_i)_{i \in \mathbb{N}}$  were assumed to be infinite, we can always find events that are smaller than a finite part of the infinitely decreasing sequence.

As all finitely consistent sets of formulae are also consistent, there exists a model  $\hat{\mathcal{E}}$  of  $\Phi \cup \text{Th}(\mathcal{E})$ . I will use  $\varepsilon, \varepsilon'$  etc. as meta-variables for elements that realize one of the types of infinitesimal objects,  $\Phi[(e_i)_{i \in \mathbb{N}}]$ .

As  $\text{Th}(\mathcal{E})$  contains only atomic sentences in  $L(\mathcal{E})$  (i.e. importantly, not the clause about the homogeneity of  $P$ ) we can moreover consistently assume that  $\neg P(\varepsilon)$  for all infinitesimal objects. (This step of the construction will be formally legitimised below by a model construction that proves its consistency.)

We can now form the set of all infinitesimal small objects below  $P$  that are too small to be  $P$  themselves:  $\lambda \varepsilon ( \exists e ( P(e) \ \& \ \varepsilon \subset e ) \ \& \ \neg P(\varepsilon) )$ . For convenient reference, let us call this area the infinitesimal part below  $P$ .

$$INF(P)(\varepsilon) \leftrightarrow \exists e ( P(e) \ \& \ \varepsilon \subset e ) \ \& \ \neg P(\varepsilon)$$

The model  $\hat{\mathcal{E}}$  hence comes up to our expectations about “zooming into” the lower end of event ontology in the following way: We maintain everything that we believed about previously recognized events ( $\mathcal{E}$  is a substructure of  $\hat{\mathcal{E}}$ ). All previous  $P$ -events as well as those that are between earlier  $P$ -events remain  $P$ -events. However, there is a lower level of previously unrecognized events that are not  $P$ .

### 3 An Example

In order to exemplify the above construction, I will repeat it on the basis of common mathematical structures. We will start with the real numbers  $\mathbf{R}$  and the set of all *open* intervals  $I$  over  $\mathbf{R}$ . Let us call this set  $\mathcal{E}$ , in order to stress that we are not supposed to consider the internal structure of the objects in question from now on.

We can now take  $\mathcal{E}$  to be the domain of events of our event structure and extend this set to a full event structure; specifically by adding the linearly ordered real numbers as our domain of time points. Let me define the basic relations and functions on events in  $E$  as follows:

- The timeline will consist of the real numbers  $(\mathbf{R}, \leq)$ .
- For all  $e$  in  $\mathcal{E}$ ,  $\tau(e) := I$  iff  $e=I$  (remembering  $e$ 's internal structure for a moment)
- For all  $e, e'$  in  $\mathcal{E}$ :  $e \oplus e'$  is defined iff  $e = ]x;y[$  and  $e' = ]w;z[$  and either  $]x;y[$  and  $]w;z[$  have nonempty intersection or  $y=w$ .  
In case (i),  $e \oplus e' :=$  the event represented by  $]x;y[ \cup ]w;z[$   
In case (ii),  $e \oplus e' := ]x;z[$   
(If desired, the operation  $\oplus$  can be made commutative)
- An event  $e$  is a mereological part of another event  $e'$ ,  $e \subset e'$  iff, seen as intervals in  $\mathbf{R}$ ,  $e \subset e'$ .

We can now choose the extension of a homogeneous predicate  $P$  in  $E$ , starting from some maximal  $P$ -event  $e$ .

$$P(e^*) \text{ iff } e^* \subset e$$

Let us check that the structure  $\mathcal{E}$  conforms to axioms (2) to (6) above:

- (3) There is no lower boundary to events:  
 $\forall e \exists e' (e' \subset e)$

This holds true, because for each interval  $I$  in  $\mathbf{R}$  there are more open intervals that are true parts of  $I$ .

- (2) Homogeneous predicates  $P$  apply to events that consist of  $P$ -parts all the way down:  
 $HOM(P) \rightarrow \forall e \forall e' (P(e) \wedge e' \subset e \rightarrow P(e'))$

This holds true due to definition of the extension of  $P$ .

- (4) Boolean Structure: There is a summation operation  $\oplus$  defined on events that adds temporally adjacent events (incl. overlapping events) to larger events:  
 $\forall e \forall e' ( \neg \exists e^* ( \tau(e) < \tau(e^*) < \tau(e') \rightarrow \exists f ( e \oplus e' = f ) ) )$

This holds true due to the definition of  $\oplus$ . Of interest to us are events with a non-intersecting temporal extension ( $]x;y[$  and  $]y;z[$  with  $y$  in the temporal

extension of neither). Here, the addition of events diverges from simple set union in  $\mathbf{R}$ .

(5) Betweenness: Between any two events, there is another one.

$$\forall e \forall e' (e' \subset e \rightarrow \exists e^* (e' \subset e^* \subset e))$$

(6) Differences: If  $e'$  is part of  $e$ , then there are non-overlapping  $e''$ ,  $e'''$  that add up  $e'$  to  $e$ :

$$\begin{aligned} & \forall e \forall e' (e' \subset e \rightarrow \\ & [\exists e'' (e' \oplus e'' = e \wedge \neg \exists e^* (e^* \subset e' \wedge e^* \subset e'')) \\ & \vee \exists e'' e''' (e' \oplus e'' \oplus e''' = e \wedge \neg \exists e^* (e^* \subset e' \& e^* \subset e'')) \wedge \neg \exists e^* (e^* \subset e' \& e^* \subset e'')] ] \end{aligned}$$

Both (5) and (6) hold true due to construction.

The construction of infinitesimal elements over this initial event structure  $\mathbf{E}$  will result in the introduction of events that would correspond to single points in  $\mathbf{R}$ . Each zero-convergent sequence  $(e_i)_{i \in \mathbf{N}}$  in  $\mathcal{E}$  corresponds to a convergent sequence of intervals in  $\mathbf{R}$ :

$$([x_i; y_i])_{i \in \mathbf{N}}, \text{ convergent series of intervals.}$$

It is a theorem in  $\mathbf{R}$  that the limes element of such sequences exist.

$\hat{\mathcal{E}}$  hence  $\hat{\mathbf{E}}$  arises from  $\mathcal{E}$  by adding (closed) intervals that consist of one point only. The closure over these will yield  $\hat{\mathcal{E}} = \{\text{all open and closed intervals over } \mathbf{R}\}$ . We can consistently assume that all events that correspond to single points in  $\mathbf{R}$  ( $[x; x]$ ) are not in the extension of  $P$ .

The property  $P$  is therefore not homogeneous in the strong sense in  $\hat{\mathcal{E}}$  that *each and any* part of a  $P$ -event is again a  $P$ -event. However, homogeneity can be stated in the following weaker form:

$$\begin{aligned} \text{HOM}(P) \leftrightarrow & [ \forall e \forall e' (P(e) \& e' \subset e \rightarrow P(e') \vee \text{INF}(P)(e')) \\ & \wedge \forall e (P(e) \rightarrow \exists e' (e' \subset e \& P(e'))) ] \end{aligned}$$

Note that the structure as it is defined so far does not support axiom (3). Even though there is no lower limit to  $P$ -events, there are smallest events that have no proper parts, namely the events that correspond to single points.

In order to obtain a structure that supports (3), the construction would need to adopt the assumption that single points in fact hide another infinity of events. A concrete structure that illustrates this step can be built on the basis of tuples

of real numbers. If we call  $E_0$  the part in  $\mathcal{E}$  below some infinitesimal event  $e$ , we can set:

$$E_0 := \{ ](x,a); (x,b)[ \mid x, a, b \in \mathbf{R} \text{ and } a \leq b \}$$

For all events  $e, e'$  in  $E_0$ :

$$e' < e : \Leftrightarrow e = ](x,a); (x,b)[ \text{ and } e' = ](x,a'); (x,b')[ \text{ and } ]a'; b'[ \subseteq ]a; b[.$$

I will not further explore whether we can faithfully assume that the temporal extension of all these events comes down to the same point in time (plausibly). If we decide that we cannot, if we in other words maintain that events have unique temporal extension, then we are forced to add infinitesimal elements to the time line as well (see Robinson 1974:244).

As an aside, I would like to mention that the initial event structure in this example appears to shed light on a paradox about time that was posed by Sebastian Löbner (p.c. in 1997). He pointed out that we have conflicting intuitions about time. On the one hand, we have a notion that there can be two immediately adjacent but nonintersecting time phases. On the other hand, we usually assume that between any two distinct time points there must be a third one, distinct from both ( i.e. density). These intuitions are in fact not both supported in the same model, in the present construction. However, this model construction can explain how we shift between two possible conceptualizations of eventualities where one view supports assumption (i) and the other supports assumption (ii).

#### 4 Solution Two: Sorites

The introduction of infinitesimally small events has turned out to be a consistent way to explain the conflicting intuitions listed in section one. However, you might object that the solution locates the “hazy phase” at the wrong point. You might maintain that the intuition that “*all walkings consist of smaller walkings*” does not come about as our failure to see small events. Let us take a closer look into this example. In fact, events for which we would cease to think about a *walking* are still quite macroscopic. Certainly, one step is not a *walking*. Certainly, two steps are not sufficient for a healthy *walking* either. Certainly, there seems to be some boundary somewhere between three and 100 steps (very loosely speaking) where the single steps end, and the real walking starts?

Looking at it from the upper end, we might likewise propose that the intuition that “*all walkings consist of smaller walkings*” comes about differently. Perhaps it means something like “if  $e$  is a walking, and if I take away one step of  $e$

to get to  $e'$ , then  $e'$  will be a walking as well". We are not able to imagine the case where a walk  $e$  minus one step  $e'$  results in something too small for a walking. Cases like these have been discussed as the *heap paradox* in logic and philosophy. We can recast it as *weak homogeneity*. Consider the following condition.

$$(WH) \quad \forall e \forall e' \forall e'' (P(e) \ \& \ e=e' \oplus e'' \rightarrow (P(e') \vee P(e'')))$$

Condition (WH) is more cautious than full homogeneity. It reflects an intuition something like "if a  $P$ -event can be subdivided into two parts, then at least one (the larger one?) is again  $P$ ". Let us assume that for each  $P$ , there is a uniform measure which distinguishes those parts of  $P$ -events that are too small to be  $P$ , e.g. 'step' for walking, 'make a sound' for 'say something' etc. Assume moreover that all ordinary  $P$ -events in any ontology consist of a finite sequence of such *STEP-P*-events. Then, by induction, we will obtain instances of sorites sequences (see Graff, 2000):

$$(14) \quad P(e)$$

$$(15) \quad \text{If } P(e) \text{ and } e=e_1 \oplus e_2 \text{ and } STEP(P)(e_2) \text{ then } P(e_1) \text{ still.}$$

$$(16) \quad \text{Any } e \text{ in } P(e) \text{ is linked to some event } \varepsilon \text{ such that there is a finite sequence} \\ e=e_1, e_2, e_3, \dots, e_k=\varepsilon \text{ where all } e_i, e_{i+1} \text{ are linked by the sorites relation in (ii), and such that } STEP(P)(\varepsilon).$$

This shows that (WH), even though it was a careful assumption about homogeneity, can not be maintained once we spell out all assumptions that are characteristic for a *heap paradox* case. Cases like these have received much discussion in the literature, and I refrain from recapitulating all the solutions that were proposed. Instead, I will base my discussion on work by Graff, specifically Graff (2000). She offers a solution to the heap paradox that rests in classical two-valued logic. This is advantageous for semantic modelling in the first case, because we need not burden semantic theory with controversial many-valued logics. More importantly, however, Graff's solution is particularly relevant to the present case insofar as it makes essential use of blind spots of the categorizing individual. Let me briefly outline her proposal.

Graff claims that, for any pair of objects (in our case: events  $e_1, e_2$ ) that are immediately linked by the sorites relation in question, the following cognitive effect occurs: Once we focus our attention on these two objects, their similarity is so salient that we cannot, subjectively, judge one to have property  $P$  but not the other. This is a subjective and essentially context-driven judgement, as Graff argues. If we decide for two events  $e_1$  and  $e_2$  where  $e_1$  is a *walking* and  $e_2$  is just one step shorter than  $e_1$  that  $e_2$  is likewise a *walking*, we tacitly expect

that the two events  $e'$  and  $e''$  which are a *walking*, a *non-walking* and separated by just one step are just somewhere lower on the scale of ever smaller events. This holds similarly for the dual case of two *non-walkings*. Globally speaking, therefore, there exists a borderline, i.e. two events  $e_1, e_2$  such that

$$\neg P(e_1) \wedge \neg P(e_2) \wedge P(e_1 \oplus e_2)$$

Looking at things locally, however—and this seems to be the kind of perspective that feeds our armchair intuitions about event ontology—we maintain principle (WH). Like for the previous solution, the condition on homogeneous predicates needs to be adapted:

$$HOM(P) \leftrightarrow [ \forall e \forall e' ( P(e) \ \& \ e' \subset e \rightarrow P(e') \vee INF(P)(e') ) ]$$

Note that in this case, we can not safely assume that all  $P$ -events have at least *some* parts that are again  $P$ . There is a strict boundary somewhere that separates  $P$  from  $INF(P)$ . We are just unable to locate it precisely:

$$\exists e ( P(e) \wedge \forall e' ( e' \subset e \rightarrow \neg P(e') ) )$$

Hence, the sorites solution and the infinitesimal construction, even though both capture our armchair intuitions about events, can be clearly distinguished by the logical truths that are supported by either kind of model.

## 5 Outlook and Summary

Both the construction of infinitesimal events and the sorites explanation appear to capture some of the essence of how we think about very small events. At present, I have no conclusive argument to favour one or the other treatment.

However, the existence of two logically distinct ways to fine-tune the notion of homogeneity could be put to work to distinguish cases that could not be differentiated by earlier theories. This is particularly interesting for cases where predicates appear to be homogenous, but are not so perceived by speakers.

For example, Zucchi and White (Zucchi 2001) investigate the so-called twigs and sequences puzzle. It has been observed that a sentence like (17) is ill-formed, although geometry tells us that the initial segment of a line is again a line and hence, each line consists of an infinity of shorter lines.

(17) \*John drew a line for 2 minutes.

The difference between a case like (17) and a (well-formed) sentence like '*John took a nap for 3 minutes*' could be located in the different ways in which

we think about smaller parts of a nap, and smaller parts of lines. For example, we could assume that objects like lines, sequences etc. are viewed as sorites-homogeneous but not infinitesimally homogeneous.

$$HOM(P) \leftrightarrow [ \forall e \forall e' ( P(e) \ \& \ e' \subset e \rightarrow P(e') \vee INF(P)(e') ) ] \\ \exists e ( P(e) \wedge \forall e' ( e' \subset e \rightarrow \neg P(e') ) )$$

Atelic predicates in the sense of aspect semantics, by contrast, could be required to be homogeneous in the strict sense.

$$HOM(P) \leftrightarrow [ \forall e \forall e' ( P(e) \ \& \ e' \subset e \rightarrow P(e') \vee INF(P)(e') ) \\ \wedge \forall e ( P(e) \rightarrow \exists e' ( e' \subset e \ \& \ P(e') ) ) ]$$

This opens up a new possible line to distinguish between *John drew a line for 2 minutes* and *John ate beans for 10 minutes*, and hence could explain their different behaviour.

To summarize, in this paper I drew attention to conflicting assumptions about the lower end of event ontology that are suggested by different linguistic phenomena. Homogeneity (as required in the modelling of aspect) suggests that some properties  $P$  apply to large events and all their smaller parts, no matter how far down we look. Minimal-event-NPIs on the other hand suggest that events can indeed be too small to count as an element in the extension of  $P$  (for the same, or similar, properties  $P$ ). I suggested that the dilemma can be resolved in two different ways.

The Infinitesimal Event construction rests on the assumption that the conceptual ‘blind spot’ of speakers that drives them to make inconsistent assumptions about event ontology on different occasions essentially consists in ignoring irrelevant material. As soon as we are forced to acknowledge the existence of extremely small events, we enrich our ontology, and readjust notions like  $HOM$  accordingly.

The starting point of the sorites solution is the hypothesis that we make idealised assumptions about the properties of very small events in everyday reasoning, just in order to keep matters simple. As soon as we are forced to think seriously about these minute eventualities, we acknowledge our idealisation as false, and readjust notions like  $HOM$  accordingly.

It appears very difficult to devise definite arguments in favour of one or the other of these two options. However, their joint existence opens up new perspectives in the investigation of aspect and related issues.

## 6 References

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